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Identity and Learning: a study on the effect of student-teacher gender matching on learning outcomes

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Identity and Learning: a study on the effect of student-teacher gender matching on learning outcomes

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Abstract

In this paper we examine whether students' and teachers' identity play any role in the learning outcome of students. Specifically, we ask if a student benefits by learning from a teacher of her same gender. Unlike the existing literature which explains such interaction through role model effect or Pygmalion effect, we explain such interaction in terms of gender based sorting behaviour across private and public schools. Our results are driven by two critical differences between male and female individuals. For male and female teachers, the difference comes from their differential transaction costs of traveling to schools at remote locations. For students, the difference between male and female members comes from the differential returns to education accrued to their parents; for girl students, a lower fraction of the return comes to their parental families as they start living with their husband's family after their marriages. These factors create a sorting pattern which makes the female teachers and students of the highest quality attend private schools in urban location. This creates a positive gender matching effect only for urban, private schools. We find support for our theoretical predictions when we test them using Young Lives Survey (YLS) data collected from Andhra Pradesh.

JEL classification: I20, J16

Keywords: Teacher-student matching, Gender identity, Education, Gender norms, India

1 Introduction

The literature on teacher-student gender matching that studies the effect of a student being matched with a teacher of same sex on the student's performance largely attributes the positive effect on two possible channels – Pygmalion effect or role model effect. In both these mechanisms, the explanations are driven by cultural beliefs (either the teacher's belief that a student of his/her same sex can do better or the student's belief that he/she can be like his/her teacher) which presumably do not vary with other socio-economic parameters. The studies which aim to estimate the gender matching effect therefore, aim for scenarios where student teachers are matched randomly so that the estimated effect of same-sex matching purely picks up the beliefs described earlier. In our paper, we take a different approach where we model a selection mechanism driven my existing gender norms in India. The mechanism ensures that high quality female teachers and students self select themselves to private schools in urban location thereby creating a positive gender matching effect.

Before going into our framework, let us briefly review the existing literature on teacher-student gender matching. The existing literature, as we have mentioned before, are based on two theoretical conjecture – *Pygmalion effect* (PE) and *Role Model Effect* (RME). The Pygmalion effect – named after the mythical Greek sculptor Pygmalion who fell in love with a statue he carved – conjectures that if a teacher expects high performance from a student, the student responds by putting up good performance, making it a self fulfilling prophecy. One of the pioneering studies that found support PE hypothesis was done by Rosenthal and Jacobson (1968). After this study was published a series of studies were done to re-examine the Pygmalion effect for students from different age and social strata. Despite the fact that non consensus emerged from these studies, PE remained one of the two most powerful hypothesis explaining gender matching effect (Braun, 1976).The other most popular candidate explanation for gender matching effect is the *Role Model Effect* where a student of certain race or sex idolizes a teacher from the same identity and gets inspired to perform better. The RME, however, is not limited to school performance and works for other decisions such as career choice as well (Almquist and Angrist, 1971; Basow and Howe, 1980).

While the existing empirical papers on teacher-student gender matching cite these two candidate explanations, we have not come across any paper that tests one channel against the other. As both the hypotheses are based on psychological factors, it is difficult to test for either of these using data which collects data on examination grades and other observable socio-economic parameters. Hence, the studies which tests for gender matching effect impliciltly assume the existence of either (or both) of the channels and then look for the effect of a student being taught by a teacher of his/her same sex. From this perspective, the major empirical challenge in this literature is to solve the selection problem as students and teachers are not always randomly assigned to a class. Many studies have tried to exploit the longitudinal data structure to solve this problem (Dee, 2007).

There is however, no clear consensus on the existence of student-teacher gender matching effects. While some studies such as Dee (2007) found positive gender matching effect for eighth grade students in the United States, some others such as Carrington et al. (2008) found little support for role model effect with eleven year old children enrolled in British schools. Conducting a study conducted from a multilevel perspective, Marsh et al. (2008) also did not find any positive effect of male teachers on male students in Australian schools. While these studies are mostly based on single country, Cho (2012) conducted a cross country study using data from OECD countries and did not find any effect of teacher-student gender matching. Similar results suggesting little or no support for gender interaction are reiterated in studies based in Ohio, United States (Price, 2010), Stockholm, Sweden (Holmlund and Sund, 2008) and Florida, United States (Egalite et al., 2015). Among these studies, Egalite et al. (2015) found mixed results – they found no effect of gender matching for the elementary school students but found some effect of modest magnitude for middle and high school students. In a qualitative study based on classroom observations and individual interviews of 7-8 year old children enrolled in British schools and their teachers, Francis et al. (2008) does not find any support for gender matching effect.

While the above mentioned studies, based on developed countries, found little evidence in favour of gender matching, some other papers, mostly focusing on developing countries, find positive gender matching effect. Rawal and Kingdon (2010), for example, examining the role of identity matching on the lines of gender, caste and religion using a data set from India, found positive significant effect of matching along all the dimensions of identity including gender. In another study conducted with a a sample of 8th graders in Chile, Paredes (2014) finds a positive effect of a match between female teachers and female students. However, they do not find any negative effect on male students who are matched with female teachers. A similar positive result of gender matching has also been found by Muralidharan and Sheth (2016) in an Indian sample and Lim and Meer (2017) in a Korean sample.

A cursory look at the discussion of the above literature reveals that the existence of gender matching effect can at best be called ambiguous. There is however, a pattern – in most of studies based on developed countries, the evidence of gender matching is weaker than their less developed counterpart. This could be the result of differential gender norms prevailing in developed and less developed countries. It is possible that the selection mechanisms that guide the teachers and students are very different in these two settings, which has implications on the extent of PE and RME in the resulting gender interactions. In our paper, we model such a mechanism based on gender norms prevailing in Indian society and test the implications of the model using a survey data.

In our theory, we argue that there are two types of norms that lead to gender based sorting mechanisms for both teachers and students. For the teachers, there exists an ideal location of residence where they want to stay because of availability of amenities such as good hospitals and good schools for their children. In India, urban centers typically host such amenities. The teachers reside in these places and travel to work places (i.e. schools) if they are far away. The real costs of long hours of traveling are higher for women teachers than their male counterpart as the gender norms prevailing in India require the women to take care of the household chores. Even in families where women's work participation is encouraged, household chores remain the primary responsibility of the women members. Such a situation makes a women teacher prefer a work place near her home. Therefore, we argue that women do not prefer government schools where hiring is done centrally and upon recruitment, candidates can be posted in far away places. The private schools with their decentralized hiring practice, on the other hand, favored by the women teachers. From the student's side, given the norm of patrilocal exogamy, where wives migrate to co-reside with their husbands' kin, parents know that after their daughters' marriage, only a small fraction of their daughters' future income will come back to them and as a result, only the highest quality girls are sent to costly private schools (The notion that patrilocal exogamy leads to worse educational outcome for women in India is also confirmed by Rammohan and Vu (2018). With these two mechanisms in place, private schools receive female teachers and femal students of high quality making a strong, positive gender interaction effect. We test these theoretical hypotheses using Young Lives Survey data collected from Andhra Pradesh, India.

In our theory, the positive effect of gender matching does not stem from any cultural belief about the quality of female teacher or student. Rather, it comes from a sorting mechanism emanating from a deep rooted gender norm in the society. In essence, our story is similar to the one discussed by Munshi and Rosenzweig (2006) where an otherwise conservative, traditional gender norms led to a good outcome in which girls opt for English education and white collar jobs. In our study also, we show that traditional norms lead to good outcome for female students. While we do not have any direct policy suggestion, we show that gender matching effect can be a result of a complex selection mechanism. Unlike experimental set up, the assignment of teachers and students in schools follow some selection mechanism and any effective policy for the female teachers or students must internalize the selection mechanism. Our paper, contributes in this area by detailing one such mechanism and providing empirical support for them.

Besides the specific literature of gender matching effect, the paper is also related to the literature of the effect of teachers' characteristics on students performance (Rockoff, 2004; Clotfelter et al., 2006; Kane et al., 2008; Metzler and Woessmann, 2012). This is also related to the papers on matching effect of other identity dimension such as race (Rezai-Rashti and Martino, 2010; Diamond et al., 2004; Egalite et al., 2015; Eddy and Easton-Brooks, 2011). The paper is organized as follows – in section 2 we present the theory and in section 3, empirical evidence followed by the conclusion.

2 Theory

2.1 Model Preliminaries

We consider a model of school choice by teachers as well as students and examine the effect of the resulting matching on the students' performances. Schools are distributed over different geographical locations. Each school employs one teacher. All teachers prefer a certain location as their residence and they stay there. If they get a job somewhere else they commute from their preferred residential location rather than shifting to their work place. We assume that that teachers prefer to live in urban centers and if they get a job in rural location they commute. The cost of commute is rising in the distance between the school location and their preferred residential location. we justify this assumption in Indian context on the ground that the level of health and other facilities are much higher than what is available in rural areas. The lack of proper health facility in rural India is often discussed in media reports and policy research (Das et al., 2012). We provide further evidence supporting the assumption of urban preference from our data in the empirical section. The students of a specific location, on the other hand, must attend a school located in that area. In other words, the cost of travel is infinitely high for the students. Given this structure, we want to study the teacher-student matching and the effect of this matching on students' performances.

2.1.1 Schools

We consider two types of schools - private and government schools. The schools are uniformly distributed over the interval $[0, 1]$ location-wise. At each point over the interval $[0, 1]$, there is a government school. Thus, the number of government schools is of measure 1. However, whether there would be a private school at a particular location is determined from the model. The existence of a private school at a particular location requires two conditions to be met – first, there must be a teacher who is willing to teach in the private school at that location at the current private school wage, and second, there must be students in that particular location who are willing to get enrolled in a private school. We denote a school's location by $x \in [0, 1]$. We assume that the teachers in government schools are better paid than teachers in private schools. In other words, $w_g > w_p$ where w_g and w_p are teachers' wages in government and private schools respectively. Barring few elite schools private school teachers in India are often under-paid with less job security. They can be fired any time and typically work without social security benefit (Muralidharan and Sundararaman, 2013). Our data, even though it does not contain the details about teachers' salary, shows that majority of the private school teachers have temporary jobs.

We also assume that the schools do not face any capacity constraint. Any student who is willing to go to a particular school in her locality gets that opportunity. However, the private schools charge a school fee of t from each student while the government schools are free. All schools try to recruit better quality teachers.

2.1.2 Teachers

Teachers are of two broad categories - Male and Female. However, within each category, there are teachers of different qualities. Within each category $i \in \{F, M\}$, teacher quality, q_i is uniformly distributed over the interval $[0, 1]$ with higher q_i indicating higher quality. For each quality, there is exactly one Male and one Female teacher. Thus, the total number of teachers is of measure 2 with measure 1 for female teachers and measure 1 for male teachers. The most preferred location for all teachers irrespective of their categories and qualities is $x = 1$. However, the cost of traveling to a distant school is different for Male and Female teachers. We assume that for a teacher of category $i \in \{F, M\}$, the pay-off from accepting a job with wage

w in a school located at x is

$$u_i(w, x) = w - \theta_i(1 - x) \tag{1}$$

where $\theta_F = 1$ and $\theta_M = \theta < 1$. The cost of traveling to a distant school is higher for Female teachers than the Male ones. This can be justified using the notion that the cost of time away from home is higher for females because their contributions in home output is relatively higher than their male counterpart. Such cost differential can also be rationalized in terms of patriarchal norms that discourages women to work outside home. We also assume that all teachers' reservation pay-off is 0.

2.1.3 Students

At each location x, there are students of two categories - Girls (f) and Boys (m). Within each category, there are students of different abilities. We assume that at each location x and for each student category j, student ability a_j is uniformly distributed over the interval $[0, 1]$. Hence at each location, there are one boy and one girl students with ability a and this is true for all $a \in [0, 1]$. Thus, the measure of students at each location is 2 with 1 for boys and 1 for girls.

The students' school choice decisions are taken by the households. We assume a student must select a school in her/his location, i.e. traveling to a distant school is prohibitively costly. So the choice is limited between the local government school and the private school if one is available in the locality. We assume that the future productivity of a student depends on the knowledge acquired at school (k) as well as her own ability (a). The knowledge is verifiable and hence the potential employers can make the payment to a student contingent on the knowledge. However, the ability of a student is private information and the quality of teacher the student interacted with is non-verifiable. The employers only know the type of school a student attended at the time of making the job offer and hence can make the wage payment contingent on the average ability of the students attending that particular type of school. Given this formulation, the relative earning of a student going to a private school vis-a-vis that of one going to a government school with the same level of knowledge is the ratio of average abilities of students attending these two types of schools, i.e. $\frac{\bar{a}_p}{\bar{a}_g}$, where $\bar{a}_l$ is the average ability of students attending a type l school.

Suppose that at the time of making the school choice decisions for their children, the households' perceived relative premium from private schooling of their kids is

$\beta \geq 1$. We will later show that in equilibrium there exists $\beta > 1$ such that $\beta = \frac{\bar{a}_p}{\bar{a}_g}$. Thus, the expected net return for a child with knowledge k from private schooling is

$$y_p(k) = \beta Ak - t \tag{2}$$

and from government school is

$$y_g(k) = Ak \tag{3}$$

where A is the marginal return to knowledge acquired from school and t is the cost of private schooling and

The families choose their children's school based on their future income that accrues to the family. In this respect, there is a critical difference between boys and girls. Given the culture of patriarchy prevailing in India, women move to her husband's ancestral home after marriage while men often stay with their parents. Hence, the expected share of future income of a student that comes back to his/her parental family is higher for boys than for girls. We model this by assuming that for boys the entire future income is expected to come back to the family while for the girls this amount is only a fraction of expected future income[1]. This distinction, in our model becomes critical when parents choose schools for their children. Hence, the net return from schooling in a private school for a boy child is given by

$$y_p^m = \beta Ak - t \tag{4}$$

For a girl child the future return to private school for the family becomes

$$y_p^f = \alpha\beta Ak - t \tag{5}$$

where α is the fraction of future return from schooling that comes back to the family for girls. Similarly, the return to education in government schools for boy and girl children are given respectively in the following equations:

$$y_g^m = Ak \tag{6}$$

For a girl child the future return to government school for the family becomes

$$y_g^f = \alpha Ak \tag{7}$$

[1]Our results go through as long as we assume that the share of future income coming back to the parental family is higher for the boys than the girl.

2.1.4 Knowledge production

We assume that students are matched with their teacher in schools and as a result knowledge is produced. The knowledge production function has two inputs - the student's ability, a, and the teacher's quality, q, and takes the following form:

$$k = aq \tag{8}$$

The marginal effect of teacher's quality on student's knowledge depends on the student's ability.

2.2 Teacher-school matching

We first analyze the school choice decision of the teachers. We assume that if a teacher accepts a job in government schools, he/she is randomly allocated to any government school in the interval $[0, 1]$ over which the government schools are spread. Therefore, ex-ante the expected location of the government school for any teacher is $\frac{1}{2}$ given the uniform distribution of the government schools. Hence, the expected pay-off from a government job is

$$\Pi_g^i = w_g - \frac{\theta_i}{2}$$

for $i = F, M$. On the other hand, if a teacher gets a job in a private school located at x, her pay-off is

$$\Pi_p^i = w_p - \theta_i (1 - x)$$

A female teacher accepts a government job over an offer from a private school at location x, if and only if

$$w_g - \frac{1}{2} \geq w_p - (1 - x)$$

This leads to the following threshold condition for accepting government jobs for female teachers

$$x \leq w_g - w_p + \frac{1}{2} = x_0 \tag{9}$$

A male teacher does the same if and only if

$$w_g - \frac{\theta}{2} \geq w_p - \theta (1 - x)$$

This leads to the location threshold for male teachers

$$x \leq \frac{w_g - w_p}{\theta} + \frac{1}{2} = x'_0 \tag{10}$$

Notice that since the private schools and government school board always try to recruit better quality teachers, the teachers of higher quality get to make their choices earlier than their low quality counterpart. In case of government school jobs however, the teachers cannot choose their exact school locations.

We now impose restrictions on the parameters to ensure that both male and female teachers are distributed over both types of school.

A1 $w_g > \frac{1}{2}$, $\theta < w_p < 1$

A2 $w_g - w_p < \frac{1}{2}$

A3 $w_g - w_p > \frac{\theta}{2}$

The restriction on w_g in A1 makes sure that the female teachers find it remunerative to accept government jobs. We will shortly see that the bounds on w_p generate voluntary unemployment for female teachers while full-employment for male teachers. In other words, these restrictions make sure that the participation constraint for the female teachers becomes binding at some point, while the same for the male teachers never binds. The implication of this assumption will become clearer later on.

A2 ensures that a female teacher prefers a job in a private school of her most preferred location ($x = 1$) over a government job and hence $x_0 < 1$. A3, on the other hand, makes sure that as long as government jobs are available, male teachers prefer government jobs over teaching in a private school.

We have already shown that female teachers prefer private schools at location $x \in (x_0, 1]$ to government school jobs. In absence of any gender bias from the employers, teachers get job offers sequentially according to their qualities. As a result, the female teachers at the top of the quality ladder ($q_F \in (x_0, 1]$) will accept offers from private schools located at $x \in (x_0, 1]$. All male teachers, on the other hand, prefer government jobs over private ones irrespective of their locations. But all male teachers do not get government jobs when female teachers compete for government jobs. As the top quality female teachers opt out of government jobs and choose private jobs at the locations $x \in (x_0, 1]$, male teachers with $q_M \in (x_0, 1]$ accept government job offers. However, these male teachers have no choice of their job locations.

Male teachers start facing competition from their female counterpart for government jobs, after the private jobs at locations $x \in (x_0, 1]$ are filled up by female teachers. Both male and female teachers prefer government jobs to private jobs at the locations $x \leq x_0$. Note that of all the government jobs – which are of measure 1 – jobs of measure $(1 - x_0)$ are already filled in by the top quality male teachers. Hence, government jobs of measure x_0 remains to be filled in and both male and female teachers compete for them. These jobs will be shared equally between male and female teachers moving downwards in the quality ladder from x_0. Thus, female teachers of quality $q_F \in [\frac{x_0}{2}, x_0]$ will now get government jobs. Note that male teachers of quality $q_M \in [x_0, 1)$ are already in government jobs and now male teachers of quality $q_M \in [\frac{x_0}{2}, x_0)$ take up government jobs. This results in male teachers of quality $q_M \in [\frac{x_0}{2}, 1)$ accepting government jobs.

In the previous paragraph, we looked at the teachers' choice between private jobs and government jobs. Once government jobs are filled-up, the rest can either choose private jobs or remain unemployed .

For female teachers, joining a private school at location x is better than remaining unemployed if and only if

$$w_p - (1 - x) \geq 0$$

This leads to

$$x \geq 1 - w_p = x_1 \tag{11}$$

Given A1, $1 - w_p > 0$, implying that $x_1 > 0$. This means that there exists some location x_1 for which female teachers prefer to remain unemployed rather than working in a private school located beyond x_1 .

For the male teachers, the condition for joining a private school at location x rather than remaining unemployed is

$$w_p - \theta (1 - x) \geq 0$$

This would imply

$$x \geq 1 - \frac{w_p}{\theta} = x_1' \tag{12}$$

Once again, A1 ensures that the above holds for every $x \geq 0$, i.e. the male teachers are willing to join a private school even at location 0 and no male teacher remain voluntarily unemployed.

Notice that since $w_g > \frac{1}{2}$, $x_1 < x_0$. Remember that female teachers prefer government schools to private schools in locations $x < x_0$. On the other hand, they rather remain unemployed than joining private schools in locations $x < x_1$. Hence, in the interval $x_1 \geq x < x_0$, their first preference is government jobs. But if they don't get one they are ready to join private schools rather than remain unemployment.

Let us now summarize our findings from above. We show that male teachers of quality $q_M \in (x_0, 1]$ get absorbed in the government schools in the first round when they face no competition from their female counterpart. In the next round of the quality ladder, both male and female teachers of quality $q \in [\frac{x_0}{2}, x_0]$ accepts government jobs. Hence, the total number of male teachers in government jobs become $(1 - \frac{x_0}{2})$ and total female teachers in government jobs become $\frac{x_0}{2}$. This exhausts the government jobs.

Now for private schools at locations $x \in [x_1, x_0]$, we are left with both male and female teachers with quality less than $\frac{x_0}{2}$ and they will be filling up the private jobs in these locations. Evidently, half of these private jobs will be filled up by people from each category. Hence, female teachers with $q_F \in [\frac{x_1}{2}, \frac{x_0}{2})$ will be employed in these private schools. This implies that for private schools, the top quality $(1 - x_0)$ female teachers accept private jobs, and then $(\frac{x_0}{2} - \frac{x_1}{2})$ of lower quality female teachers also accept private jobs. Among the male teachers $\frac{x_0}{2}$ accept private jobs. This makes the total measure of teacher willing to get employed in private jobs equal to $(1 - \frac{x_1}{2})$. Hence, $\frac{x_1}{2}$ jobs remain vacant. But all the male teachers are willing to employed, and because these jobs are beyond x_1, female teachers prefer to remain unemployed to joining these remote location private schools. We assume that the only input needed to run a school is a teacher. We have shown that the private schools located at $x \in [0, \frac{x_1}{2})$ run into a supply bottleneck in the sense that these cannot get a teacher to run the school and thus cannot survive.

We summarize the above observations in the following figures. The first two figures show the quality-wise distribution of female and male teachers among government and private schools, while the last one shows the teacher profile of the private schools in different locations.

Figure 1, 2 and 3 here

In the next section, we model the students' schools choice. For that, we need the average teacher quality in government schools as student calculate their pay-offs from attaining government schools from that. For private schools they know the exact teacher quality in a location.

2.3 Students' school choice decisions

Each household decides the type of school[2] for its ward considering the net future return from education. The household, while making the choice, distinguish between boys and girls because it believes that while the whole future earning of a boy accrues to the family, only a fraction, α, of that the family can retain for a girl.

Since all male teachers with quality $q_M \in \left[\frac{x_0}{2}, 1\right]$ and all female teachers with quality $q_F \in \left[\frac{x_0}{2}, x_0\right]$ work in government schools, the average quality of all teachers in government schools can be easily determined as[3]

$$\bar{q}^g = \frac{2 + x_0^2}{4}.$$

If a student with ability a is sent to the government school in the locality, the expected acquired knowledge would be

$$k_g(a) = a\bar{q}^g$$

If the same student is sent to the private school, acquired knowledge depends on the student's location which is the same as the school's location. If the student's location is x, then

$$k_p(a, x) = \begin{cases} ax & \forall x \in (x_0, 1] \\ a.\frac{x}{2} & \forall x \in (x_1, x_0] \\ a.\left(x - \frac{x_1}{2}\right) & \forall x \in \left[\frac{x_1}{2}, x_1\right] \end{cases}$$

[2]In this paper, the choice is limited to the local government and private schools given our assumption of prohibitive transport cost for the students. The quality of the local private school - essentially the teacher quality - is endogenously determined from the model. In a separate paper (Bhattacharya et al 2020), we modeled household choice of school quality when government and private schools of differing qualities are available and showed how the choice of school type as well as school quality depends on household characteristics.

[3]It is easy to see that

$$\begin{aligned} \bar{q}^g &= \left(1 - \frac{x_0}{2}\right)\bar{q}_M^g + \left(x_0 - \frac{x_0}{2}\right)\bar{q}_F^g \qquad (13) \\ &= \left(1 - \frac{x_0}{2}\right)\left(\frac{x_0 + 2}{4}\right) + \frac{x_0}{2}\frac{3x_0}{4} \\ &= \frac{4 - x_0^2 + 3x_0^2}{8} \\ &= \frac{2 + x_0^2}{4} \end{aligned}$$

For locations $x < \frac{x_1}{2}$, the private schools cannot sustain because of teacher unavailability.

Now consider the households' school choice decision about a boy student of ability a located at $x \in (x_0, 1]$. If this boy is sent to a government school, his expected future earning would be

$$y_g^m(a,x) = Aa\bar{q}^g$$

If he is sent to a private school, his net expected earning is

$$y_p^m(a,x) = \beta Aax - t$$

The boy is sent to the private school if and only if

$$y_p^m(a,x) \geq y_g^m(a,x)$$

$$\Leftrightarrow \beta Aax - t \geq Aa\bar{q}^g$$

$$\Leftrightarrow a \geq \frac{\frac{t}{A}}{\beta x - \bar{q}^g} = a_1^m(x,\beta) \tag{14}$$

A girl at the same location will be sent to a private school if and only if

$$y_p^f(a,x) \geq y_g^f(a,x)$$

$$\Leftrightarrow \alpha\beta Aax - t \geq \alpha Aa\bar{q}^g$$

$$\Leftrightarrow a \geq \frac{\frac{t}{\alpha A}}{\beta x - \bar{q}^g} = a_1^f(x,\beta) \tag{15}$$

Similarly, for every location $x \in (x_1, x_0]$ and $x \in (\frac{x_1}{2}, x_1]$, we can find the critical ability levels for boys and girls above which they are sent to private schools. We denote these $a_2^i(x,\beta)$ and $a_3^i(x,\beta), i = f, m$ respectively and these can be derived as

$$a_2^m(x,\beta) = \frac{\frac{t}{A}}{\beta\frac{x}{2} - \bar{q}^g} \tag{16}$$

$$a_2^f(x,\beta) = \frac{\frac{t}{\alpha A}}{\beta\frac{x}{2} - \bar{q}^g} \tag{17}$$

and

$$a_3^m(x,\beta) = \frac{\frac{t}{A}}{\beta\left(x - \frac{x_1}{2}\right) - \bar{q}^g} \tag{18}$$

$$a_3^f(x,\beta) = \frac{\frac{t}{\alpha A}}{\beta\left(x - \frac{x_1}{2}\right) - \bar{q}^g} \tag{19}$$

Notice that for all x, β, $a_1^m(x,\beta) < a_1^f(x,\beta)$.

$a_j^i(x,\beta)$ falls with x as well as β for all i and j. Thus, higher the perceived return from private schooling relative to government schooling, higher is the number of students put to private school in every location where a private school exists. Similarly, given β, the more remote the private school is, the lower is the quality of teacher and hence lower is the return to private schooling. Thus, remote private schools would have lower number of students relative to government schools.

For locations $x \in (x_1, x_0]$ and for $\beta \geq 1$ the general conditions for sending boys and girls to private schools are detailed in the next two equations:

$$a_2^m(x,\beta) \leq 1 \Leftrightarrow x \geq \frac{\frac{t}{A} + \bar{q}^g}{\frac{\beta}{2}} = x^{2m}(\beta) \tag{20}$$

and

$$a_2^f(x,\beta) \leq 1 \Leftrightarrow x \geq \frac{\frac{t}{\alpha A} + \bar{q}^g}{\frac{\beta}{2}} = x^{2f}(\beta) \tag{21}$$

For locations $x \in (\frac{x_1}{2}, x_1]$, these conditions are

$$a_3^m(x,\beta) \leq 1 \Leftrightarrow x \geq \frac{\frac{t}{A} + \bar{q}^g}{\beta} + \frac{x_1}{2} = x^{3m}(\beta) \tag{22}$$

and

$$a_3^f(x,\beta) \leq 1 \Leftrightarrow x \geq \frac{\frac{t}{\alpha A} + \bar{q}^g}{\beta} + \frac{x_1}{2} = x^{3f}(\beta) \tag{23}$$

For any given x, $a_j^i(x,\beta)$ for all $i \in \{m,f\}$, $j \in \{1,2,3\}$ falls with β. Thus, in any given location, more students are sent to private school as the perceived return rises. Moreover, $x^{ji}(\beta)$ for all $i \in \{m,f\}$, $j \in \{2,3\}$ also falls with β implying that students in more locations are sent to private schools as β rises.

2.4 Finding equilibrium

We find the equilibrium in terms of β. For every β, the set of students going to private and government schools at every location is uniquely determined. This in turn determines the average abilities of students over all locations going to private and government schools, $\vec{a}^p$ and $\vec{a}^g$, as functions of β. We look for $\beta^* > 1$ such that

$$\beta^* = \frac{\bar{a}^p(\beta^*)}{\bar{a}^g(\beta^*)} \tag{24}$$

Parent's school choice for their children is driven by their objective of future income maximization. The reason for choosing one type of school over the other is driven by the differences in the returns from a specific type of school. Private school premium comes from two components – relative earning of private school students vis-a-vis government school students and teacher's quality in private schools vis-a-vis government schools. The first component is captured by β while the second component is captured by q – quality of the teachers. For private schools, teacher's quality is location specific and rational parents, this being a full information model, can predict the teacher's quality in the private school in their location. For government schools, however, teachers are randomly posted and parents therefore take the average teacher's quality for government schools in all locations. Of these two parameters, β is endogenously determined in the equilibrium. Teacher's quality on the other hand – both for government and private – is exogenously given to the parents.

It is important to note that at the location $x = x_0$ teacher's quality in private schools takes a discontinuous plunge. This is because of the fact, that till the location x_0, female teachers prefer joining private schools to government schools. Consequently, the top quality female teachers are employed at private schools in these locations. Top male teachers prefer joining government schools, but they are randomly posted across locations. Hence, government schools in the locations $x \in (x_0, 1]$ does not necessarily get the best teachers. After the private jobs at $x \in (x_0, 1]$ are exhausted female teachers start joining government jobs along with their male counterpart. This continues until the government jobs are exhausted. After that female teachers start accepting private jobs at locations x_0 and below. The lowest quality female teachers who accepted private jobs at $x \in (x_0, 1]$ is x_0, while the best teacher – who can be either male or female– at the location $x \in (x_1, x_0]$ is $\frac{x_0}{2}$. This creates a discontinuity in terms of the teacher quality at the point x_0.

Hence, in our quest for equilibrium we start by looking at the school choice decision at the location x_0 . At $\beta = 1$, private schools have no advantage in terms of wage premium. Thus, the difference between return from a private and that from a government school, if any, comes from the differences at the teacher's quality in these two types of schools.

We assume that $a_1^m(x_0, 1) \leq 1$ i.e. even if there is no perceived private school premium, some students at $x = x_0$ are sent to the private school. The private school at $x = x_0$ has a teacher of quality x_0 while the government school at the same location has a randomly allocated teacher. Therefore, students would be sent to the private school at x_0 only if the quality of the private school teacher at x_0 exceeds that of the average government school teacher by an amount that justifies the private school fee[4]. This is assumed in A4.

A4 $\frac{t}{\alpha A} \leq x_0 - \bar{q}^g$

Given A4, at $\beta = 1$,

$$a_1^m(x_0, 1) \leq a_1^f(x_0, 1) < 1$$

Hence, some boys are girls[5] are sent to private schools at $x = x_0$.

We have already shown that at every location $x \leq x_0$, the private school teacher quality is less than $\frac{x_0}{2}$. Since average government school quality $\bar{q}^g = \frac{2+x_0^2}{4}$ is greater than $\frac{x_0}{2}$, in these locations children are not sent to private schools in absence of any private school premium.

Now suppose β increases from 1. If β rises above $\frac{\frac{t}{A}+\bar{q}^g}{\frac{x_0}{2}}$ so that $x^{2m}(\beta) \leq x_0$, households start sending boys to private schools in the locations $x \in [x^{2m}(\beta), x_0]$. Once $\beta \geq \frac{\frac{t}{\alpha A}+\bar{q}^g}{\frac{x_0}{2}}$, the girls in the locations $x \in \left[x^{2f}(\beta), x_0\right]$ will be sent to private schools. As β rises above $\frac{\frac{t}{A}+\bar{q}^g}{\frac{x_1}{2}}$, boys and then eventually girls at locations in the range $\left(\frac{x_1}{2}, x_1\right)$ are sent to private schools.

We can now characterize the distribution of students of each gender at all locations among private and government schools and hence average ability of students going to private and government schools for different values of β. It is fairly straightforward to verify that there exists at least one[6] finite $\beta^* > 1$ at which the perceived private school premium is exactly equal to the relative average ability of the private school students, i.e.

$$\beta^* = \frac{\bar{a}^p(\beta^*)}{\bar{a}^g(\beta^*)}$$

We have relegated the proof of existence of the equilibrium to appendix. The equilibrium private school premium, β^*, depends on the parameters of the model.

[4]A necessary condition for this assumption to hold is $x_0 > \bar{q}_g = \frac{2+x_0^2}{4}$ which requires x_0 to be high enough. For this, the difference between the teacher salaries in government and private schools needs to be high

[5]We can relax this assumption and our results will remain qualitatively unaffected as long as $\frac{t}{A} < 1 - \bar{q}^g$. Given this condition, we will always find an equilibrium in which private schools exist and the best students are always sent to private schools generating a positive labour market premium for private school goers. However, A4 makes the exposition clearer without compromising on basic message that we attempt to convey here.

[6]There may be more than one equilibrium.

2.5 Main results

We are now in a position to discuss the main results of the paper. The locations at which the private schools would have students depend on β^*. Notice that if a private school gets students at location y, then all private schools at location $x \in (y, 1]$ would also have students. Suppose $\underline{x}(\beta^*)$ is the remotest location at which a private school can get students. If $\beta^* \leq \frac{\frac{t}{A}+\bar{q}^g}{\frac{x_0}{2}}$, then only private schools at locations $x \in [x_0, 1]$ would have students since for every $x < x_0$, $a_2^m(x, \beta) > 1$. Thus for this range of β^*, $\underline{x}(\beta^*) = x_0$. Similarly, for other values of β^*, we can identify $\underline{x}(\beta^*)$ in the following manner:

$$\underline{x}(\beta^*) = \begin{cases} x_0 & \forall \beta^* \in \left(1, \frac{\frac{t}{A}+\bar{q}^g}{\frac{x_0}{2}}\right] \\ x^{2m}(\beta^*) & \forall \beta^* \in \left(\frac{\frac{t}{A}+\bar{q}^g}{\frac{x_0}{2}}, \frac{\frac{t}{A}+\bar{q}^g}{\frac{x_1}{2}}\right] \\ x^{3m}(\beta^*) & \forall \beta^* \in \left(\frac{\frac{t}{A}+\bar{q}^g}{\frac{x_1}{2}}, \infty\right) \end{cases} \tag{25}$$

Since for any finite β^*, $\underline{x}(\beta^*) > \frac{x_1}{2}$, even though there are some teachers willing to accept jobs in private schools at all locations $x \geq \frac{x_1}{2}$, these schools cannot survive because of lack of students. This leads to involuntary unemployment among teachers. The female teachers do not accept employment in private schools located at $x < x_1$. The male teachers however are willing to work at private schools located at $x \geq \frac{x_1}{2}$. Thus, the involuntary unemployment among male teachers are higher than the female teachers. These are stated in the following proposition.

Proposition 1 *In equilibrium, there is involuntary unemployment among teachers. The extent of involuntary unemployment is higher among male teachers than among female teachers at equilibrium.*

Interestingly, the standard remedy of involuntary unemployment - wage cut - may aggravate the problem instead of curing it. If w_p goes down, x_0 will increase leading to an increase in $\bar{q}_g$. This makes the government schools more attractive to students at all locations and as a result in some locations where the private school were getting students may not get them any more. This would tend to aggravate the problem of unemployment.

We next discuss gender-wise ability distribution of students in different types of school. First, notice that at every location at which a private school exists, ability wise top students from both male and female categories go to private schools while the rest goes to government school. Thus, at every location the average ability of students from each category going to private school exceeds the average ability of their counterparts going to the government school. At every $x > \underline{x}(\beta^*)$, the male students with ability $a \in [a^m(x, \beta^*), 1]$ attend private school while those with ability $a \in [0, a^m(x, \beta^*))$ go to government school. Thus, at every $x > \underline{x}(\beta^*)$, the average ability of male students going to private school is $\frac{a^m(x,\beta^*)+1}{2}$, while that of male students going to government school is $\frac{a^m(x,\beta^*)}{2}$. For the female students, these are $\frac{a^f(x,\beta^*)+1}{2}$ and $\frac{a^f(x,\beta^*)}{2}$ for private and government schools respectively. These observations lead to our next proposition.

Proposition 2 *The average abilities of both female and male students going to private schools exceed the average qualities of the same category students going to government schools at every location.*

We can now discuss our main results regarding gender-wise student performance in private schools. The number of female students going to private schools as well as their abilities depend among other things the equilibrium private school premium. We derive our results for the case $\beta^* \in \left(\frac{\frac{t}{A}+\bar{q}^g}{\frac{x_0}{2}}, \frac{\frac{t}{A}+\bar{q}^g}{\frac{x_1}{2}} \right]$, for which $\underline{x}(\beta^*) = x^{2m}(\beta^*) \in [x_1, x_0)$. However, the results are robust across different equilibrium values of β^* for which cut-off locations are specified in equations (25). Given this β^*, at each location $x \in [x^{2m}(\beta^*), x_0)$, the abilities of male private school goers are $a \in [a_2^m(x, \beta^*), 1]$. At the locations $x \in [x_0, 1]$, the male students with abilities $a \in [a_1^m(x, \beta^*), 1]$ go to private schools.

Since at each school all students are being taught by the same teacher, the average performance of the male students going to a particular school is determined by the average ability of the male students in that particular school and the quality of the teacher. Thus for any private school at location x, the average performance of male students is given by

$$P^m(x,\beta^*) = \begin{cases} \frac{1+a_2^m(x,\beta^*)}{2}.\frac{x}{2} & \forall\ x \in [x^{2m}(\beta^*), x_0) \\ \frac{1+a_1^m(x,\beta^*)}{2}.x & \forall\ x \in [x_0, 1] \end{cases} \tag{26}$$

since the teacher quality in $x \in [x^{2m}(\beta^*), x_0)$ is $\frac{x}{2}$ while the teacher quality in $x \in [x_0, 1]$ is x.

The number of male students going to private schools at location x is given by

$$n^m(x, \beta^*) = \begin{cases} 1 - a_2^m(x, \beta^*) & \forall\ x \in [x^{2m}(\beta^*), x_0) \\ 1 - a_1^m(x, \beta^*) & \forall\ x \in [x_0, 1] \end{cases} \tag{27}$$

The average performance of all male students in private schools can thus be computed as

$$\bar{k}_p^m = \frac{1}{N_m(\beta^*)} \int_{x^{2m}(\beta^*)}^{1} n^m(x,\beta^*)\, P^m(x,\beta^*)\, dx \tag{28}$$

where

$$N_m(\beta^*) = \int_{x^{2m}(\beta^*)}^{1} n^m(x,\beta^*)\, dx$$

The average performance of all female students in private schools is

$$\bar{k}_p^f = \frac{1}{N_f(\beta^*)} \int_{x^{2f}(\beta^*)}^{1} n^f(x,\beta^*)\, P^f(x,\beta^*)\, dx \tag{29}$$

where $n^f(x,\beta^*)$, $P^f(x,\beta^*)$, N_f and $x^{2f}(\beta^*)$ defined accordingly.

The comparative performance of girls vis-a-vis the boys in private schools is stated in our next proposition.

Proposition 3 *Suppose A1-A4 hold. The average performance of girls exceeds that of boys in private schools.*

The proof of the proposition is technical and relegated to appendix.

We next discuss the performances of the boys and girls in private schools when they are matched with teachers of different genders. First, consider the private schools at locations $x \in [x_0, 1]$. The students in these schools are being taught by only female teachers. In the private schools at locations $x \in [x^{2m}(\beta^*), x_0)$, half the teachers are male and the rest are female. Hence any student going to a private school at these locations, will be taught by a female teacher with probability $\frac{1}{2}$ and by a male teacher by probability $\frac{1}{2}$.

Let us now consider the performance of the boys. The average performance of the boys in private schools when matched with female teachers can be derived exactly as in Eq. (28) except that for schools located at $x \in [x^{2m}(\beta^*), x_0)$, we have to use $\frac{1-a_2^m(x,\beta^*)}{2}$ instead of $1 - a_2^m(x,\beta^*)$ since each school would have a female teacher with probability $\frac{1}{2}$. Thus, the average performance of the boys in private schools when matched with female teachers can be written as

$$\bar{k}_{pF}^m = \frac{\int_{x^{2m}(\beta^*)}^{x_0} \frac{\left(1-a_2^m(x,\beta^*)\right)}{2} . \frac{1+a_2^m(x,\beta^*)}{2} . \frac{x}{2} dx + \int_{x_0}^{1} \left(1 - a_1^m(x,\beta^*)\right) . \frac{1+a_1^m(x,\beta^*)}{2} .x dx}{\int_{x^{2m}(\beta^*)}^{x_0} \frac{\left(1-a_2^m(x,\beta^*)\right)}{2} dx + \int_{x_0}^{1} \left(1 - a_1^m(x,\beta^*)\right) dx}$$

For notational convenience we write

$$\bar{k}_{pF}^m = \frac{\frac{I_2^m}{2} + I_1^m}{\frac{N_2^m}{2} + N_1^m}$$

where I_j^t and N_j^i, $j = 1, 2, i = m.f$ are defined as in Eqs. (??), (??), (??) and (??) in Appendix. The average performance of boys when matched with male teachers can be written as

$$\bar{k}_{pM}^m = \frac{\int_{x^{2m}(\beta^*)}^{x_0} \frac{\left(1-a_2^m(x,\beta^*)\right)}{2} \cdot \frac{1+a_2^m(x,\beta^*)}{2} \cdot \frac{x}{2} dx}{\int_{x^{2m}(\beta^*)}^{x_0} \frac{\left(1-a_2^m(x,\beta^*)\right)}{2} dx} = \frac{\frac{I_2^m}{2}}{\frac{N_2^m}{2}}$$

It is easy to verify that

$$\frac{I_1^m}{N_1^m} > \frac{1 + a_1^m (1, \beta^*)}{2}.x_0$$

while

$$\frac{I_2^m}{N_2^m} < \frac{x_0}{2}$$

since both $a_1^m (x, \beta^*)$ and $a_2^m (x, \beta^*)$ are falling in x and $a_2^m (x^{2m} (\beta^*) , \beta^*) = 1$ by definition. Because $a_1^m (1, \beta^*) > 0$,

$$\frac{1 + a_1^m (1, \beta^*)}{2}.x_0 > \frac{x_0}{2}$$

and hence

$$\frac{I_1^m}{N_1^m} > \frac{I_2^m}{N_2^m}$$

Therefore,

$$\frac{\frac{I_2^m}{2} + I_1^m}{\frac{N_2^m}{2} + N_1^m} > \frac{\frac{I_2^m}{2}}{\frac{N_2^m}{2}}$$

holds. A similar result can be obtained for girls as well. This is stated in our next proposition.

Proposition 4 *Suppose A1-A4 hold. Both boys and girls in private schools perform better on average when matched with a female teacher than when matched with a male teacher.*

Since the average quality of female teachers is higher than that of male teachers in private schools, the intuition behind the result is straightforward.

We next explore whether there is any difference in performance of the boys and girls of same ability in private schools. Consider a boy with ability a. If $a < a_1^m (1, \beta^*)$, this boy is never sent to a private school wherever he is located. If $a \in [a_1^m (1, \beta^*) , a_1^m (x_0, \beta^*))$, he is sent to a private school only if he is located at x such that $a_1^m(x,\beta^*) \leq a$. Similarly, if $a \in [a_1^m(x_0,\beta^*), a_2^m(x_0,\beta^*))$, the same boy would be sent to private school only if he is located at $x \in [x_0, 1]$. If $a \geq a_2^m(x_0,\beta^*)$, he would be sent to private schools at locations x such that $a_2^m(x,\beta^*) \leq a$. Similarly, we can trace out the cut-off locations for girls for every ability. However, for the boys and girls of same ability, the cut-off location for the girls are generally above that of the boys since $a_i^m(x,\beta^*) < a_i^f(x,\beta^*)$. However, if[7] $a \in [a_1^f(x_0,\beta^*), a_2^m(x_0,\beta^*))$, the cut-off location for both boys and girls is x_0.

Suppose for ability a, we denote the cut-off location for boys by $x_m(a)$ and girls by $x_f(a)$. Then,

$$x_m(a) = \begin{cases} \frac{\frac{t}{aA}+\bar{q}_g}{\beta} & \forall\ a \in [a_1^m(1,\beta^*), a_1^m(x_0,\beta^*)) \\ x_0 & \forall\ a \in [a_1^m(x_0,\beta^*), a_2^m(x_0,\beta^*)) \\ \frac{\frac{t}{aA}+\bar{q}_g}{\frac{\beta}{2}} & \forall\ a \in [a_2^m(x_0,\beta^*)), 1] \end{cases}$$

and

$$x_f(a) = \begin{cases} \frac{\frac{t}{\alpha aA}+\bar{q}_g}{\beta} & \forall\ a \in [a_1^f(1,\beta^*), a_1^f(x_0,\beta^*)) \\ x_0 & \forall\ a \in [a_1^f(x_0,\beta^*)), a_2^f(x_0,\beta^*)) \\ \frac{\frac{t}{\alpha aA}+\bar{q}_g}{\frac{\beta}{2}} & \forall\ a \in [a_2^f(x_0,\beta^*)), 1] \end{cases}$$

If α is not very low, the critical ability levels of the boys and girls can be easily ranked. We assume that α is such that the following holds:

$$a_1^m(1,\beta^*) < a_1^f(1,\beta^*) < a_1^m(x_0,\beta^*) < a_1^f(x_0,\beta^*) < a_2^m(x_0,\beta^*) < a_2^f(x_0,\beta^*) < 1$$

For ability levels $a \in [a_1^m(1,\beta^*), a_1^f(1,\beta^*))$, only boys are sent to private schools and these boys are exclusively taught by female teachers. For any other a, both boys and girls are sent to private schools.

Notice that except for $a \in [a_1^f(x_0,\beta^*), a_2^m(x_0,\beta^*))$, $x_m(a) < x_f(a)$. Consider $a \in [a_1^f(1,\beta^*), a_1^f(x_0,\beta^*))$. The expected performance of a boy with ability a is

$$\frac{1}{1-x_m(a)}\int_{x_m(a)}^{1} axdx = \frac{a(1+x_m(a))}{2}$$

while that of a girl with same ability is

$$\frac{1}{1-x_f(a)}\int_{x_f(a)}^{1} axdx = \frac{a(1+x_f(a))}{2}$$

[7] We are assuming $a_1^f(x_0,\beta^*) < a_2^m(x_0,\beta^*)$ which will hold if α is not very small.

Since $x_f(a) > x_m(a)$ at these levels of a, the expected performance of a girls with ability a will be better than a boy with same ability. If $a \in [a_1^f(x_0, \beta^*)), a_2^m(x_0, \beta^*))$, $x_m(a) = x_f(a) = x_0$ and hence the boys and girls would perform similarly. If $a \in [a_2^m(x_0, \beta^*), a_2^f(x_0, \beta^*))$, $x_m(a) < x_0$ while $x_f(a) = x_0$. In this case, the expected performance of a boy is

$$\begin{aligned} & \frac{1}{1 - x_m(a)} \left[\int_{x_m(a)}^{x_0} a.\frac{x}{2} dx + \int_{x_0}^{1} axdx \right] \\ = & \frac{a}{1 - x_m(a)} \left[\frac{1}{2} - \frac{x_0^2}{4} - \frac{(x_m(a))^2}{4} \right] \end{aligned}$$

while that of a girl is

$$\frac{1}{1 - x_0} \int_{x_0}^{1} axdx = \frac{a}{1 - x_0} \left[\frac{1}{2} - \frac{x_0^2}{2} \right] = \frac{a(1 + x_0)}{2}$$

It is easy to verify that

$$\frac{(1 + x_0)}{2} > \frac{1}{1 - x_m(a)} \left[\frac{1}{2} - \frac{x_0^2}{4} - \frac{(x_m(a))^2}{4} \right]$$

for all $x_m(a) < x_0$. Finally, for $a \geq a_2^f(x_0, \beta^*)$, we can show that

$$\frac{a}{1 - x_m(a)} \left[\frac{1}{2} - \frac{x_0^2}{4} - \frac{(x_m(a))^2}{4} \right] < \frac{a}{1 - x_f(a)} \left[\frac{1}{2} - \frac{x_0^2}{4} - \frac{(x_f(a))^2}{4} \right]$$

for $x_f(a) > x_m(a)$. These are reported in our next proposition.

Proposition 5 *Suppose A1-A4 hold. Among the girls and boys who are sent to private school a girl is expected to perform generally better than a boy with the same ability.*

Our final result compares how students of different genders but same ability fare when matched with teachers of different gender. First consider a student of ability a. Notice that the girls in private schools $a < a_2^f(x_0, \beta^*)$ are not taught by by male teachers at all, we cannot judge the relative performance of male and female teachers in teaching girls with ability lower than $a_2^f(x_0, \beta^*)$. We thus consider $a \geq a_2^f(x_0, \beta^*)$. The girls of ability a are taught by female teachers at locations $[x_0, 1]$, while at locations $[x_f(a), x_0)$ they are taught by a female teacher with probability half and by a male teacher with probability $\frac{1}{2}$. Thus, the expected performance of a girl conditional on being matched with a male teacher is

$$k_f^M(a) = \frac{1}{\frac{1}{2}(x_0 - x_f(a))} \int_{x_m(a)}^{x_0} \frac{1}{2} a. \frac{x}{2} dx = \frac{a}{2} \frac{x_f(a) + x_0}{2}$$

Similarly, the the expected performance of a girl with same a conditional on being matched with a female teacher is

$$\begin{aligned} k_f^F(a) &= \frac{1}{\frac{1}{2}(x_0 - x_f(a)) + 1 - x_0} \left[\int_{x_m(a)}^{x_0} \frac{1}{2} a. \frac{x}{2} dx + \int_{x_0}^{1} a.x dx \right] \\ &= \frac{a}{2} . \frac{1 - \frac{3x_0^2}{4} - \frac{(x_f(a))^2}{4}}{1 - \frac{x_0}{2} - \frac{x_f(a)}{2}} \end{aligned}$$

For a boy with ability $a \geq a_2^f(x_0, \beta^*)$, the expected performances are

$$k_m^M(a) = \frac{a}{2} \frac{x_m(a) + x_0}{2}$$

and

$$k_m^F(a) = \frac{a}{2} . \frac{1 - \frac{3x_0^2}{4} - \frac{(x_m(a))^2}{4}}{1 - \frac{x_0}{2} - \frac{x_m(a)}{2}}$$

One can easily verify that $k_f^F(a) > k_f^M(a)$ and $k_m^F(a) > k_m^M(a)$. So both boys and girls perform better under female teachers than under male teachers. However, it is interesting to note that the extent of loss in performance for a boy from being matched with a male teacher rather than a female teacher is less than that of a girl of same ability. This is stated in our next proposition.

Proposition 6 *The expected performances of boys and girls of any given ability is lower under male teachers than under female teachers. However, the extent of loss is lower for the boys than for the girls.*

The result is driven by the fact that girls of any given ability get better quality teachers than the boys of the same ability on an average. This along with the fact that the average quality of female teachers is higher than the male teachers would mean that the girls lose more from being matched with a female teacher. We relegate the formal proof of the second part to appendix.

3 Empirical Analysis

3.1 Data

The data that we use for this study comes from the Young Lives school survey (YLSS) conducted in 2016-17 which supplements Young Lives Survey (YLS) which was conducted between 2002 and 2016. The YLSS survey collects data regarding the effectiveness of secondary schools in Andhra Pradesh, India. The original YLS is an international study of childhood poverty where two cohorts were surveyed – the younger cohort, (YC) who were 1 year old in 2002 and the older cohort (OC) comprising children who were 8 years old in 2002 . Children falling in both the cohorts were surveyed in 5 consecutive rounds conducted in 2002, 2006, 2009, 2013 and 2016. The school survey, YLSS, was conducted in 2016 which surveyed the secondary school students. The survey design closely followed the sample design for YC and collect data from schools where many of the YC students go (Rolleston and Moore, 2018). The sites for both YLS and YLSS are the same and they were selected from three different agro-climatic areas[8] with districts and sites being ranked according to a number of development indicators(Kumra, 2008).The administrative sub-districts (mandals)[9] are the primary sampling units in our sample. [10].

The YLSS collected data on school effectiveness using three outcome measures: Class 9 students' performance in maths, functional English, and transferable skills. The students' performance data was collected twice – at the beginning of class 9 (Wave 1) and at the end of class 9 (Wave 2). Background information on students and school characteristics were collected using students' questionnaire and school survey questionnaire respectively. The total number of students surveyed is 8355 which are spread across 7 regions in 205 schools. There are four types of schools covered in their survey – government (in this case only state government), private aided, private unaided and tribal/social welfare schools. The details for the survey design for YLSS can be found in Rolleston and Moore (2018).

[8]Coastal Andhra, Rayalseema and Telangana (Young Lives 2007)

[9]Andhra Pradesh is divided into 23 administrative districts that are further subdivided into mandals. In total there are 1125 mandals and 27000 villages in Andhra Pradesh(Kumra, 2008)

[10]The data that support the findings of this study are openly available in https://www.younglives-india.org/access-our-data

3.2 Empirical Strategy

Our model predicts that the better quality female students and female teachers self-select themselves in urban private schools following two separate mechanisms. We have made three key institutional assumptions about the private schools in our model that drive the result – higher student's fees and lower teacher's salary in private schools compared to the government ones and decentralized hiring in private schools. In the context of Andhra Pradesh there exists heterogeneity across private schools and our assumptions do not fit with all types of private schools. In Andhra Pradesh, there are four type of schools – government, private aided, private unaided and social/tribal welfare schools. We show that between two types of private schools, it is the category of unaided private school that matches the assumptions made in our model.

Among the four types of school, private unaided schools are completely dependent on student fee for running their operations while other three get government aid. As a result, the fee is the highest for *unaided private schools*. We report the fee structures in these four different types of schools in subsection 3.4. In terms of recruitment strategy, there has been considerable heterogeneity as well. According to our model, our result holds for schools where the hiring is school specific so that a candidate for a teaching job knows the location of her workplace, if hired. In terms of recruitment process there are four practices: hiring by the district/mandal educational officer, hiring by state education department, hiring by central/national government, hiring by school management or school chain management. We call the last category *local hiring* which works in a decentralized way and we expect female teachers with higher quality to self-select themselves in this type of schools. Given the preference for urban locations along with these two self-selection mechanisms, we expect our theoretical result of gender matching to hold in urban, non-aided private schools with local hiring.

3.3 Empirical Model

In our empirical specification we try to estimate of the effect of a girl student being matched with a female teacher on her test score in Wave 2. Importantly, we are able to control for her score in Wave 1 which controls for individual level heterogeneity that remains fixed over time. In this framework, we can isolate the value added learning effect of the gender match. In our empirical model, we regress mathematics test score on the interaction between the teacher's and the student's sex controlling for a host of student level, household level, teacher level and school level controls. Specifically, we estimate the following model

$$Y_i = \alpha_0 + \alpha_1 D_i^{FS} + \alpha_2 D_i^{FT} + \alpha_3 D_i^{FT} * D_i^{FS} + \beta X_i^S + \gamma X_i^T + \epsilon_i \tag{30}$$

Where Y_i = Standardized z score in Z score from mathematics test of the student i.

$D_i^{FS} = 1$ if student i is female

$D_i^{FT} = 1$ if student i's mathematics teacher is female

X_i^S = Set of control variables that captures background information of the student including household size, past test scores(score obtained in period 1) to control for their innate ability, wealth index of household, education of the caregiver, religion, whether the household faced any recent shock, whether there is any household support for the student, region.

Our main parameter of interest is α_3 that measures the interaction effect of female student and female teacher. Given the preference for urban residence and the selection mechanisms in place, we expect the sign of this parameter to be positive.

3.4 Descriptive Statistics

Before proceeding to the next section where we report the estimation, in this subsection we try to paint a general picture of the schools, students and teachers using the descriptive statistics. In our theory and estimation, the type of schools plays an important role. In table 1, which is taken from Rolleston and Moore (2018), we present the type of schools surveyed in YLSS across different districts.

Table 1: Distribution of different school-types across districts.

The table above shows that the schools are fairly distributed across different districts. The four most critical assumptions in our model are urban locational preference for teachers and high fees, decentralized hiring and low salary in private schools. In our model, we assumed that the most preferred residential location for the teachers is the urban centers. Therefore, if they get jobs in rural school they commute from their urban residences rather than stay at the school locations. We tried to justify this assumption by citing anecdotal evidence and literature which show that there is a severe lack of health and other facilities in rural India making urban centers a more preferred choice for living. In this section, we justify this assumption using data from our data set.

Table 2: Residential choice of teachers

In table 2 we show that for rural schools, only 24% of the teachers stay in the same village where the schools is , while the rest 76% choose to commute. For urban schools however, 88% of the teachers stay in the city where the school is located with only 12% commuting from outside. This clearly shows how the residential preference is heavily biased in favor of urban locations.

Let us now discuss the underlying assumption driving the selection mechanism. While the selection mechanism for women teachers depend on the hiring practice and salary, the selection mechanism for the girl students depend on the fee structure – the higher is the fees, the higher is the chance the higher ability girl students are sent to private schools. Table 3 shows that the fee is the highest for the unaided private schools which further suggests that the selection mechanism for the girl students will be strongest for unaided private schools in our setting.

Table 3: Fee structure for different school types

Let us now look at the hiring practice of teachers across school types. In figure 4 we describe the division between local and centralized hiring among the different types of school.

Figure 4 here

From this figure we clearly see support for our assumption. The incidence of local hiring is higher for private schools, but specifically so for unaided private schools. The reason is straightforward. For the aided private schools, government pays a large part of their salary and hence, keeps a tab on their hiring practices. Therefore, hiring for the aided private schools are at least, supervised by the government and done in a centralized way. Hence, private unaided schools are the schools where the interviewees know for sure which school he/she will work for, if he/she gets the job. These are the schools where the selection mechanism for teachers predicted by our model works.

Another critical assumption that drives our result is the lower salary in private schools compared to the government ones. In the next table we examine the validity of this assumption with respect to our data. The teacher's salary varies a lot even within the same type of school. In order to summarize the information in a meaningful way, in YLSS, teacher's salary is categorized in 8 groups and each group is assigned a value from 1 to 8, the lowest category being 1 and the highest being 8.

Table 5: Salary categories

These assigned values are then used to calculate the average salary for different schools groups. The result is reported in table 5 where the results are consistent with our assumptions. The salary is the lowest for the private unaided schools. In monetary terms, this is approximately between Rs.10000-20000 per month for private unaided schools. For both government and private aided schools this is approximately around Rs. 40000.

Table 5: Average salary value for Mathematics teachers

The discussion above conclusively shows that the assumptions underlying the selection mechanism in our theoretical model hold for the private unaided schools. Hence, we test our hypothesis regarding gender matching for the students in private unaided schools. As mentioned earlier, the theory will hold for urban schools, hence we further restrict our results for the urban schools.(

Before going into the estimation results, let us look at the descriptive statistics regarding the students and their family characteristics represented in table 6. In the table there are three types of variables listed in three panels. Her mathematics Z scores for waves 1 and 2 are listed in panel A. In panel B, we present school characteristics while in panel C, we enlist the household characteristics of the student. From the school information, we find that almost half the students (56% to be exact) are girls. There is also a question for the teachers which asks if they believe that boys do better than girls in studies. People who answers yes are given value 1 in the dummy variable we call male bias. We will use this information in subsection 5 to check the possible existence of Pygmalion effect. We see from the table that around 9% teachers have male bias. We also look at the variable which tells us if the teachers' birth place is same from the location of their school and we see that for 20% teachers this is the case. We also see that around half of the students attend private tuition for mathematics and the average class size in schools is 43.

Table 6: Descriptive Statistics

3.5 Results

In this section, we present our main regression results. In the subsection on empirical model we mentioned that to test our hypothesis we would regress the mathematics z score on sex dummies for teachers and students, their interaction and other controls. Our theory predicts that the female teacher-female student interaction will be positive for unaided, private schools with local hiring. We report the results in table 7. In column 1, use the main regressors i.e. teacher's sex dummy, student's sex dummy (both take the value of 1 if they are female, 0 otherwise) and their interaction along with only one control i.e. the Mathematics z score of the student in Wave 1. We find that, the coefficient for the female student is negative and significant, while the coefficient for female teacher is positive but not significant. In column 2, we add the school characteristics as control to along with the Mathematics Z score in Wave 1. The school characteristics include a dummy capturing whether a teacher's birthplace is the same as the school's location to indicate his/her locational preference and class size. We don't find much change in the coefficients.

Table 7: Effect of teacher student gender interaction in urban,unaided private schools with local hiring

In column 3, we add parental characteristics that may have an effect on the selection mechanism of the student. These controls include if the student's mother is alive, if mother can read, mother's education, if student's father is alive, if father can read and the father's education. The coefficients do not change much in size and sign – the interaction remains positive and significant. In the last column, we add other characteristics related to the student's learning environment which are mostly related to her household's conditions. We call these other household characteristics which include household asset index, household size, number of rooms at home, if there is a study place at home, if the student attended private tuition for mathematics , the number of books at home, use of computer at home and language spoken at home. The coefficient for the interaction remains more or less the same in size and it is also positive and significant.

Next, we do the same for the government schools with non-local hiring where we do not expect any effect of such gender matching. The regression results are reported in the next table. In this case, posting of the hired candidates is likely to be decided by the central hiring authority and therefore, for this type of schools location does not ex-post play any role in the selection mechanism of the teachers. We take all schools under this category, rural and urban alike. We present the results in table 8.

Table 8: Effect of teacher-student gender interaction on students' grades for government schools with non-local hiring

We find that the interaction is consistently negative and significant. However, in this case, we find that the student and teacher's sex dummy is also positive and significant throughout.

3.6 Pygmalion Effect

Let us now look at the estimation result. We run the full specification regression that we did at the previous subsection and report the results in table 9. Similar to our previous tables we only show the variables of interest and do not report the coefficients of the control variables in the main table. In column 1, we have only the male bias and it interaction with female student dummy. We find that while teachers with male bias is bad for all students, there is no differential effect for the female students in a statistically significant way. The coefficient of the interaction term is negative but not significant. In column 2, we control for our selection bias mechanism and include the female teacher-female student interaction term. That term remains positive significant, but the interaction between teacher's male bias and female student dummy remains insignificant. However, this can very well be the case that that, given that around 19% of the teachers of urban, unaided, private schools have male bias, there are very few cases of female students being matched with such a teacher. This can cause the coefficients to be insignificant. If we compare the selection bias coefficient with that reported in table 7 we find that the magnitude has gone down after taking the male bias control. However, there has been no change in the sign and level of significance. it is therefore possible that our selection bias term is partly picking up the Pygmalion effect as well.

Table 9: Reverse Pygmalion Effect here

4 Conclusion

Gender matching effect is a well researched area and found in many countries. The theoretical understanding of such a phenomenon tells us that such a matching effect is the result of either Pygmalion or role-model effect. Therefore, the main empirical challenge for estimating such an effect is to filter out the selection bias. Because both PE and RME claim that such effect is purely psychological and therefore, should not depend on other socio-economic characteristics of either the teacher or the student.

Both PE and RME can ofcourse work for a narrower identity than the gender one such as African American female or Dalit male teachers becoming a role-model for African American female or Dalit male students only. But in that case we will not find the pure gender matching effect. The pure gender matching that comes from PE or RME can be identified if the teacher-student matching process is truly random.

In our paper instead, we propose a third process which can generate such a matching. This process is based on a selection mechanism which is grounded in the economic framework of systematic gender norms. The driving forces of the selection mechanism are the two sets of gender norms which prevail in India. The first one restricts the movements of women teachers making them choose a workplace close to their home and the second one discourages the parents to send their girl children for expensive education unless they are really good ones. Separately, these two norms work against womans cause. But together, at least in our settings, they led to the matching of good teachers and good students in a particular type of schools which brings out the best from these students. There is no direct policy implication of our findings. However, the ability cut-off for the female students, depends on several socio-economic parameters such as cost of education, return to education and social norm. Norms are difficult to change, but if parameters such as cost of education and return to education can be changed differentially for women this would perhaps allow more students to join costly, private schools.

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Figures

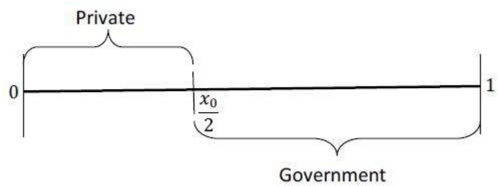

Figure 1: Quality-wise distribution of male teachers among government and private schools

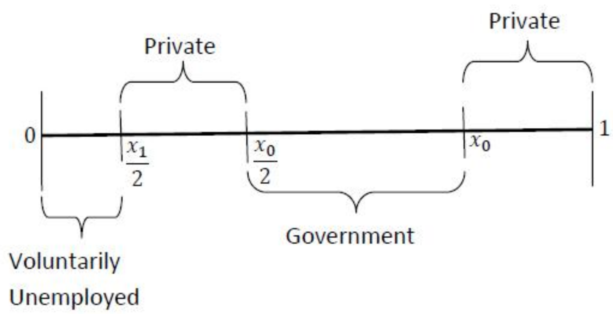

Figure 2: Quality-wise distribution of female teachers among government and private schools

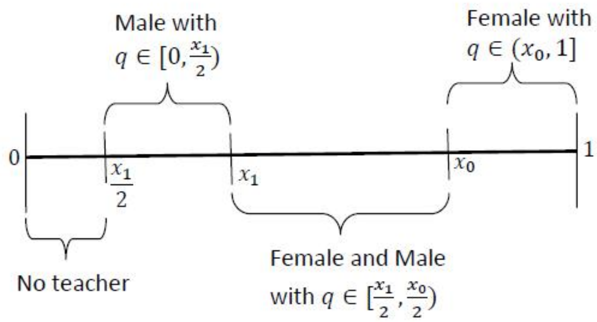

Figure 3: Teacher quality in private schools at different locations

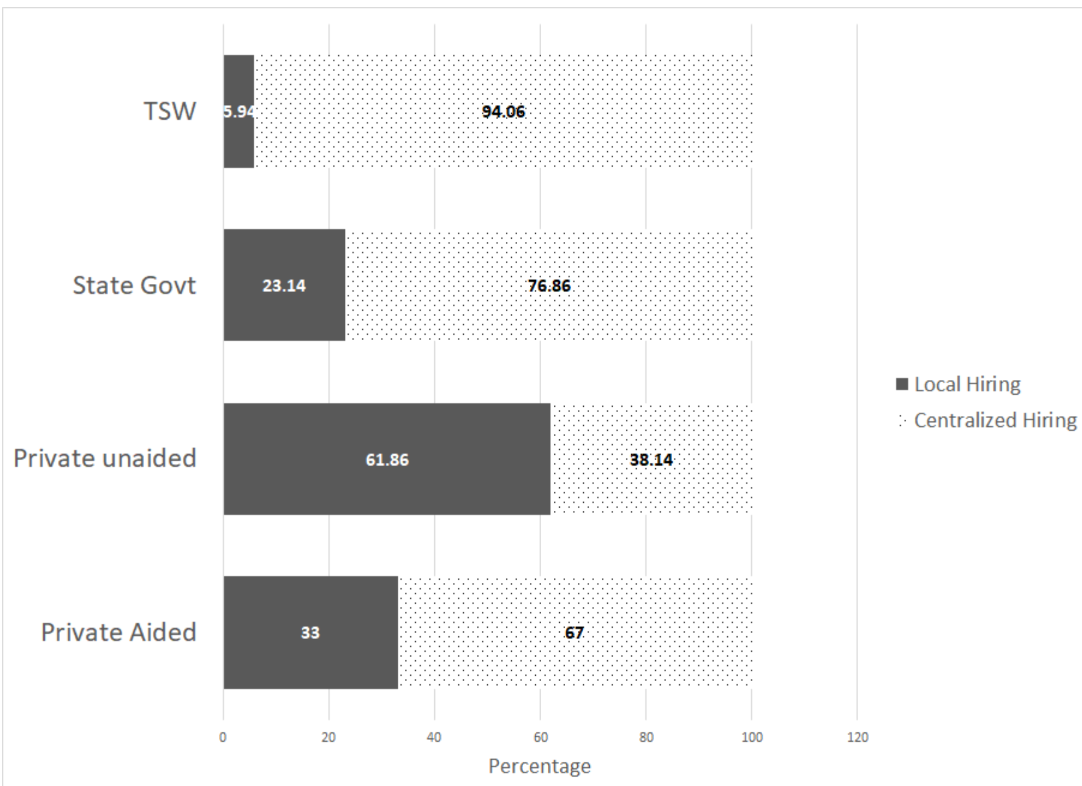

Figure 4: Distribution of hiring practice across school-types

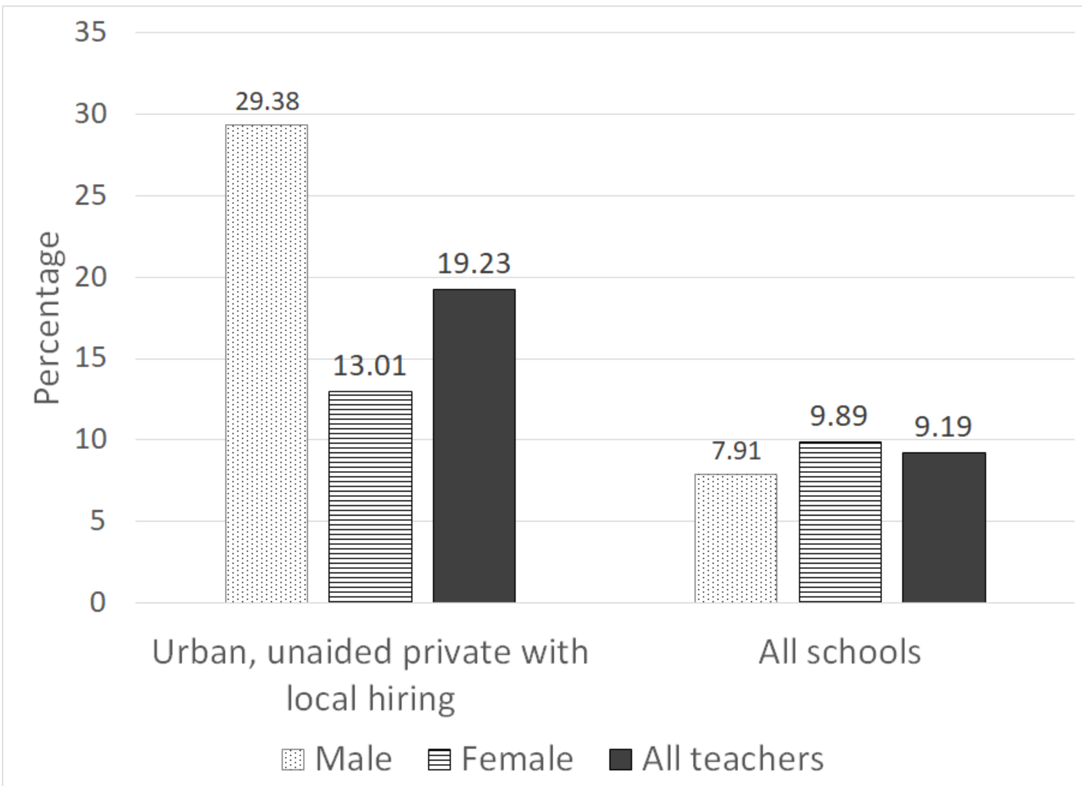

Figure 5: Distribution of male bias

Tables

Table 1: **Distribution of different school-types across districts**.

		School Types					Number of Teachers	Number of Students in Wave 1 and 2
District	**No.of Young Lives sites**	**Private Aided**	**Private unaided**	**State Government**	**Tribal/Social Welfare**	**Total Schools**		
West Godavari	2	8	5	5	11	29	77	1395
Srikakulam	5	1	9	23	17	50	128	2540
Kadapa	2	0	2	8	0	10	25	253
Anantapur	4	6	8	17	1	32	86	1360
Karimnagar	2	6	9	9	3	27	54	798
Mahbubnagar	4	0	6	19	4	29	70	993
Hyderabad	1	8	16	4	0	28	79	1016
Total	20	29	55	85	36	205	519	8355

Table 2: Residential choice of teachers

Residence of the teachers	Rural Schools	Urban Schools
The village / town where this school is	24.24	88.07
Another village/town in the mandal wher	36.3	4.73
Another village / town in the district	39.25	5.8
Another village / town in the state where this school is	0.21	1.4

Table 3: **School fee across school types**

School Type	**School Fees**				
	N	Mean	SD	Min	Max
Private Aided	1400	2447.541	4287.273	0	13000
Private unaided	2103	15972.57	8657.888	1500	43000
Government	4397	179.7728	1128.928	0	10000
TSW	1870	1.336898	8.067988	0	50

Table 4: **Salary categories**

Salary Category	Salary Value
<Rs. 5,000	1
Rs. 5,000 - 10,000	2
Rs. 10,001 - 20,000	3
Rs. 20,001 - 40,000	4
Rs. 40,001 - 60,000	5
Rs. 60,001 - 80,000	6
Rs. 80,001 - 1,00,000	7
> Rs. 1,00,000	8

Table 5: **Average salary value for Mathematics teachers**

School type	Average salary value for Mathematics teachers
Private aided	4.928571
Private unaided	3.017111
State government	4.946565
TSW	4.084123

Table 6: **Descriptive Statistics**

Variable	Obs	Mean	SD	Min	Max
A. Grades					
Standardised Math Scores: Wave-2	8183	0.00000	1	-2.51809	2.64051
Standardised Math Scores: Wave-1	9574	0.00000	1	-2.65895	3.40039
B. School Characteristics					
Female Student	9820	0.56181	0.49619	0	1
Male Bias	9820	0.09185	0.288834	0	1
Tacher birth place same as the school location	9820	0.21915	0.413688	0	1
Attended Private Tuition- Maths	9820	0.29420	0.455703	0	1
Size of the section	9820	43.12100	15.7019	6	87
C. Household characteristics					
Mom Alive	9729	0.97687	0.150314	0	1
Mom can read	9717	0.99691	0.927703	0	3
Mother's education level	9714	1.81377	1.87886	0	6
Dad Alive	9711	0.92545	0.262685	0	1
Dad can read	9720	1.13385	0.86398	0	3
Dad's education level	9705	2.44019	1.98151	0	6
HH Assets	9739	0.00000	1.84919	-3.88609	5.37668
Household size	9693	5.15413	2.8066	1	125
No. of rooms	9731	3.00678	1.18285	0	5
Study place at home	9709	0.76898	0.421509	0	1
No. of books at home	9714	1.22009	1.11583	0	4
Use of Computer	9683	0.43003	1.10301	0	4
Lang. spoken at home	9820	1.42536	1.35238	1	8

Table 7: **Gender matching effect: private schools with local hiring**

VARIABLES	(1) Baseline	(2) + School	(3) + Parents	(4) + Household
Female student	-0.172*	-0.172*	-0.211**	-0.138
	(0.0920)	(0.0984)	(0.0981)	(0.0960)
Female teacher	0.0204	-0.00868	-0.0107	-0.0284
	(0.0898)	(0.0926)	(0.0925)	(0.0908)
Female teacher*female student	0.338***	0.360***	0.362***	0.312***
	(0.114)	(0.121)	(0.120)	(0.117)
Observations	963	963	955	948
R-squared	0.372	0.376	0.402	0.447

Standard errors in parentheses
$***p < 0.01, **p < 0.05, *p < 0.1$

Table 8: **Gender matching effect: government schools with non-local hiring**

VARIABLES	(1) Baseline	(2) + School	(3) + Parents	(4) + Household
Female student	0.240***	0.224***	0.212***	0.214***
	(0.0659)	(0.0659)	(0.0681)	(0.0684)
Female teacher	0.430***	0.401***	0.388***	0.354***
	(0.0636)	(0.0636)	(0.0660)	(0.0660)
Female teacher*female student	-0.325***	-0.315***	-0.302***	-0.302***
Observations	2,655	2,655	2,593	2,557
R-squared	0.385	0.395	0.404	0.418

Standard errors in parentheses
$***p < 0.01, **p < 0.05, *p < 0.1$

Table 9: **Reverse Pygmalion effect**

	(1)	(2)
VARIABLES	Only Male bias	Bias vs Selection
Female student	0.110*	-0.0640
	(0.0583)	(0.0985)
Female teacher	0.0976*	-0.0554
	(0.0565)	(0.0897)
Female teacher*female student		0.254**
		(0.116)
Teacher's male bias	-0.457***	-0.440***
	(0.138)	(0.138)
Teacher's male bias*Female student	-0.204	-0.189
	(0.141)	(0.141)

Standard errors in parentheses
$***p < 0.01, **p < 0.05, *p < 0.1$

Appendix

4.1 Existence of Equilibrium

First notice that both $\bar{a}_p$ and $\bar{a}_g$ are continuous functions of β. By A4 some boys are sent to private school at location x_0 even when $\beta = 1$. Now suppose the number of boys and girls sent to private school at some β are $N_m(\beta)$ and $N_f(\beta)$ respectively. The farthest location at which boys and girls are sent to private schools are

$$\underline{x}_m(\beta) = \begin{cases} x_0 & \forall\ \beta \in \left(1, \frac{\frac{t}{A}+\bar{q}^g}{\frac{x_0}{2}}\right] \\ x^{2m}(\beta) & \forall\ \beta \in \left(\frac{\frac{t}{A}+\bar{q}^g}{\frac{x_0}{2}}, \frac{\frac{t}{A}+\bar{q}^g}{\frac{x_1}{2}}\right] \\ x^{3m}(\beta) & \forall\ \beta \in \left(\frac{\frac{t}{A}+\bar{q}^g}{\frac{x_1}{2}}, \infty\right) \end{cases}$$

and

$$\underline{x}_f(\beta) = \begin{cases} x_0 & \forall\ \beta \in \left(1, \frac{\frac{t}{\alpha A}+\bar{q}^g}{\frac{x_0}{2}}\right] \\ x^{2f}(\beta) & \forall\ \beta \in \left(\frac{\frac{t}{\alpha A}+\bar{q}^g}{\frac{x_0}{2}}, \frac{\frac{t}{\alpha A}+\bar{q}^g}{\frac{x_1}{2}}\right] \\ x^{3f}(\beta) & \forall\ \beta \in \left(\frac{\frac{t}{\alpha A}+\bar{q}^g}{\frac{x_1}{2}}, \infty\right) \end{cases}$$

respectively. Hence,

$$N_m(\beta) = \begin{cases} \int_{x_0}^{1} \left(1 - a_1^m(x,\beta)\right) dx & \forall\ \beta \in \left(1, \frac{\frac{t}{A}+\bar{q}^g}{\frac{x_0}{2}}\right] \\ \begin{bmatrix} \int_{x^{2m}(\beta)}^{x_0} \left(1 - a_2^m(x,\beta)\right) dx \\ +\int_{x_0}^{1} \left(1 - a_1^m(x,\beta)\right) dx \end{bmatrix} & \forall\ \beta \in \left(\frac{\frac{t}{A}+\bar{q}^g}{\frac{x_0}{2}}, \frac{\frac{t}{A}+\bar{q}^g}{\frac{x_1}{2}}\right] \\ \begin{bmatrix} \int_{x^{3m}(\beta)}^{x_1} \left(1 - a_3^m(x,\beta)\right) dx \\ +\int_{x_1}^{x_0} \left(1 - a_2^m(x,\beta)\right) dx \\ +\int_{x_0}^{1} \left(1 - a_1^m(x,\beta)\right) dx \end{bmatrix} & \forall\ \beta \in \left(\frac{\frac{t}{A}+\bar{q}^g}{\frac{x_1}{2}}, \infty\right) \end{cases}$$

Notice that as $\beta \to \frac{\frac{t}{A}+\bar{q}^g}{\frac{x_0}{2}}$, $x^{2m}(\beta) \to x_0$ and $a_2^m(x_0,\beta) \to 1$ and as $\beta \to \frac{\frac{t}{A}+\bar{q}^g}{\frac{x_1}{2}}$, $x^{3m}(\beta) \to x_1$ and $a_3^m(x_1,\beta) \to 1$. Thus, $N_m(\beta)$ is continuous in β. Similarly, we can argue that $N_f(\beta)$ is also continuous in β.

The average ability of boys going to private schools can thus derived by

$$\bar{a}_m^p(\beta) = \begin{cases} \frac{1}{N_m(\beta)}\left[\int_{x_0}^{1}\left(1-a_1^m(x,\beta)\right)\left(\frac{1+a_1^m(x,\beta)}{2}\right)dx\right] & \forall\ \beta \in \left(1, \frac{\frac{t}{A}+\bar{q}^g}{\frac{x_0}{2}}\right] \\ \frac{1}{N_m(\beta)}\left[\begin{array}{c} \int_{x^{2m}(\beta)}^{x_0}\left(1-a_2^m(x,\beta)\right)\left(\frac{1+a_2^m(x,\beta)}{2}\right)dx \\ +\int_{x_0}^{1}\left(1-a_1^m(x,\beta)\right)\left(\frac{1+a_1^m(x,\beta)}{2}\right)dx \end{array}\right] & \forall\ \beta \in \left(\frac{\frac{t}{A}+\bar{q}^g}{\frac{x_0}{2}}, \frac{\frac{t}{A}+\bar{q}^g}{\frac{x_1}{2}}\right] \\ \frac{1}{N_m(\beta)}\left[\begin{array}{c} \int_{x^{3m}(\beta)}^{x_1}\left(1-a_3^m(x,\beta)\right)\left(\frac{1+a_3^m(x,\beta)}{2}\right)dx \\ +\int_{x_1}^{x0}\left(1-a_2^m(x,\beta)\right)\left(\frac{1+a_2^m(x,\beta)}{2}\right)dx \\ +\int_{x_0}^{1}\left(1-a_1^m(x,\beta)\right)\left(\frac{1+a_1^m(x,\beta)}{2}\right)dx \end{array}\right] & \forall\ \beta \in \left(\frac{\frac{t}{A}+\bar{q}^g}{\frac{x_1}{2}}, \infty\right) \end{cases}$$

By the same argument, we made above $\bar{a}_m^p(\beta)$ is continuous in β and so is $\bar{a}_f^p(\beta)$. Now, the average productivity of all students going to private schools at all locations,

$$\bar{a}^p(\beta) = \frac{N_m(\beta)}{N_m(\beta)+N_f(\beta)}\bar{a}_m^p(\beta) + \frac{N_f(\beta)}{N_m(\beta)+N_f(\beta)}\bar{a}_f^p(\beta)$$

also continuous in β.

Remember that average ability of all students in all locations is $\frac{1}{2}$ and the total measure of students is 2. Out of these, $N_m(\beta)+N_f(\beta)$ go to private schools and the rest go to private schools. Since the overall average ability is the weighted average of the abilities of students in private and government schools with the weights being the shares of students in two types of schools, we can write

$$\frac{2-(N_m(\beta)+N_f(\beta))}{2}\bar{a}^g(\beta) + \frac{N_m(\beta)+N_f(\beta)}{2}\bar{a}^p(\beta) = \frac{1}{2}$$

Thus,

$$\bar{a}^g(\beta) = \frac{1-(N_m(\beta)+N_f(\beta))\,\bar{a}^p(\beta)}{2-(N_m(\beta)+N_f(\beta))}$$

is also continuous in β.

Since only students from the top end of the ability profile at any location go to private schools at any β, generally $\bar{a}^p(\beta) > \bar{a}^g(\beta)$. Since some students go to private schools even at $\beta = 1$,

$$\frac{\bar{a}^p(1)}{\bar{a}^g(1)} > 1$$

However, as $\beta \to \infty$, $\underline{x}_m(\beta) = x^{3m}(\beta) \to \frac{x_1}{2}$ and $\underline{x}_f(\beta) = x^{3f}(\beta) \to \frac{x_1}{2}$. Moreover, at every x, $a_i^m(x,\beta)$ and $a_i^f(x,\beta)$ converge to 0. Thus, as as $\beta \to \infty$, all students at all locations where private schools exist (private schools cannot exist at locations below $\frac{x_1}{2}$ because of lack of supply of teachers) go to private schools. Thus as $\beta \to \infty$, $\bar{a}^p(\beta) \to \frac{1}{2}$. Hence, $\bar{a}^g(\beta) \to \frac{1}{2}$ as well. Hence,

$$\lim_{\beta \to \infty} \frac{\bar{a}^p(\beta)}{\bar{a}^g(\beta)} = 1$$

Since $\frac{\bar{a}^p(\beta)}{\bar{a}^g(\beta)}$ is continuous in β, $\frac{\bar{a}^p(\beta)}{\bar{a}^g(\beta)} > 1$ at $\beta = 1$ while $\frac{\bar{a}^p(\beta)}{\bar{a}^g(\beta)} \to 1$ as $\beta \to \infty$, there must exist a $\beta^* > 1$ such that at $\beta = \beta^*$

$$\frac{\bar{a}^p(\beta)}{\bar{a}^g(\beta)} = \beta$$

This proves the existence of an equilibrium β^*.

4.2 Proof of Proposition 6

The difference in the performances of the boys and girls originates from the difference in the relative private school fees they have to bear - $\frac{t}{A}$ for boys and $\frac{t}{\alpha A}$ for the girls with $\alpha < 1$. We show that as the effective school fee rises for any particular group, average performance for that group rises at any given β^*. Suppose the effective fee for the boys is $\tau = \frac{t}{a}$. We show that $\frac{\delta \bar{k}_p^m}{\delta \tau} > 0$.

Notice that

$$\begin{aligned} \frac{\delta \bar{k}_p^m}{\delta \tau} &= \int_{x^{2m}(\beta^*)}^{1} \left[P^m(x,\beta^*) \frac{\delta}{\delta \tau} \left(\frac{n^m(x,\beta^*)}{N_m(\beta^*)} \right) + \frac{n^m(x,\beta^*)}{N_m(\beta^*)} \frac{\delta}{\delta \tau} \left(P^m(x,\beta^*) \right) \right] dx \\ & -P^m\left(x^{2m}(\beta^*),\beta^*\right) \frac{n^m\left(x^{2m}(\beta^*),\beta^*\right)}{N_m(\beta^*)} . \frac{\delta}{\delta \tau}\left(x^{2m}(\beta^*)\right) \end{aligned}$$

Since $n^m\left(x^{2m}(\beta^*),\beta^*\right) = 1 - a_2^m\left(x^{2m}(\beta^*),\beta^*\right) = 0$ by definition of $x^{2m}(\beta^*)$,

$$\frac{\delta \bar{k}_p^m}{\delta \tau} = \int_{x^{2m}(\beta^*)}^{1} \left[P^m(x,\beta^*) \frac{\delta}{\delta \tau} \left(\frac{n^m(x,\beta^*)}{N_m(\beta^*)} \right) + \frac{n^m(x,\beta^*)}{N_m(\beta^*)} \frac{\delta}{\delta \tau} \left(P^m(x,\beta^*) \right) \right] dx$$

Since $P^m(x,\beta^*)$ and $n^m(x,\beta^*)$ are only piecewise continuous, we have to integrate them separately over the two mutually exclusive intervals $[x^{2m}(\beta^*), x_0)$ and $[x_0, 1]$.

Notice that

$$\frac{1}{\frac{n^m(x,\beta^*)}{N_m(\beta^*)}} . \frac{\delta}{\delta \tau} \left(\frac{n^m(x,\beta^*)}{N_m(\beta^*)} \right) = \frac{1}{n^m(x,\beta^*)} . \frac{\delta}{\delta \tau} \left(n^m(x,\beta^*) \right) - \frac{1}{N_m(\beta^*)} \frac{\delta}{\delta \tau} \left(N_m(\beta^*) \right)$$

First consider the interval $[x^{2m}(\beta^*), x_0)$. In this interval, $n^m(x,\beta^*) = 1 - a_2^m(x,\beta^*)$ and by Eq. (16) $a_2^m(x,\beta^*) = \frac{\tau}{\beta^* \frac{x}{w} - \bar{q}^g}$. Thus,

$$\frac{\delta}{\delta\tau}\left(n^m(x,\beta^*)\right) = -\frac{\delta}{\delta\tau}\left(a_2^m(x,\beta^*)\right) = -\frac{1}{\tau}a_2^m(x,\beta^*)$$

Similarly, for the interval $[x_0, 1]$,

$$\frac{\delta}{\delta\tau}\left(n^m(x,\beta^*)\right) = -\frac{\delta}{\delta\tau}\left(a_1^m(x,\beta^*)\right) = -\frac{1}{\tau}a_1^m(x,\beta^*)$$

Since

$$N_m(\beta^*) = \int_{x^{2m}(\beta^*)}^{x_0}\left[1 - a_2^m(x,\beta^*)\right]dx + \int_{x_0}^{1}\left[1 - a_1^m(x,\beta^*)\right]dx$$

we can write

$$\begin{aligned}\frac{\delta}{\delta\tau}\left(N_m(\beta^*)\right) &= -\frac{1}{\tau}\int_{x^{2m}(\beta^*)}^{x_0} a_2^m(x,\beta^*)\,dx - \frac{1}{\tau}\int_{x_0}^{1} a_1^m(x,\beta^*)\,dx \\ &\quad - \left[1 - a_2^m\left(x^{2m}(\beta^*), \beta^*\right)\right]\frac{\delta}{\delta\tau}\left(x^{2m}(\beta^*)\right) \\ &= -\frac{1}{\tau}\left[\int_{x^{2m}(\beta^*)}^{x_0} a_2^m(x,\beta^*)\,dx + \int_{x_0}^{1} a_1^m(x,\beta^*)\,dx\right]\end{aligned}$$

where the last term vanishes because $a_2^m\left(x^{2m}(\beta^*), \beta^*\right) = 1$. However,

$$N_m(\beta^*) = 1 - x^{2m}(\beta^*) - \left[\int_{x^{2m}(\beta^*)}^{x_0} a_2^m(x,\beta^*)\,dx + \int_{x_0}^{1} a_1^m(x,\beta^*)\,dx\right]$$

and hence,

$$\frac{\delta}{\delta\tau}\left(N_m(\beta^*)\right) = -\frac{1}{\tau}\left[1 - x^{2m}(\beta^*) - N_m(\beta^*)\right]$$

Therefore, using the expressions for $n^m(x,\beta^*)$ at different intervals and some manipulations we can write

$$\frac{\delta}{\delta\tau}\left(\frac{n^m(x,\beta^*)}{N_m(\beta^*)}\right) = \begin{cases} \frac{1}{\tau}\frac{1-a_2^m(x,\beta^*)}{N_m(\beta^*)}\left[-\frac{a_2^m(x,\beta^*)}{1-a_2^m(x,\beta^*)} + \frac{1-x^{2m}(\beta^*)-N_m(\beta^*)}{N_m(\beta^*)}\right] & \text{if } x \in [x^{2m}(\beta^*), x_0) \\ \frac{1}{\tau}\frac{1-a_1^m(x,\beta^*)}{N_m(\beta^*)}\left[-\frac{a_1^m(x,\beta^*)}{1-a_1^m(x,\beta^*)} + \frac{1-x^{2m}(\beta^*)-N_m(\beta^*)}{N_m(\beta^*)}\right] & \text{if } x \in [x_0, 1] \end{cases}$$

From Eq. (26), we know that

$$\frac{\delta}{\delta\tau}\left(P^m(x,\beta^*)\right) = \begin{cases} \frac{1}{\tau}\frac{a_2^m(x,\beta^*)}{2}.\frac{x}{2} & \text{if } x \in [x^{2m}(\beta^*), x_0) \\ \frac{1}{\tau}\frac{a_1^m(x,\beta^*)}{2}.\frac{x}{2} & \text{if } x \in [x_0, 1] \end{cases}$$

Thus,

$$\begin{aligned}
\frac{\delta \bar{k}_p^m}{\delta \tau} &= \frac{1}{\tau}\int_{x^{2m}(\beta^*)}^{x_0}\left(\begin{array}{c} \frac{1-a_2^m(x,\beta^*)}{N_m(\beta^*)}\left[-\frac{a_2^m(x,\beta^*)}{1-a_2^m(x,\beta^*)}+\frac{1-x^{2m}(\beta^*)-N_m(\beta^*)}{N_m(\beta^*)}\right]\frac{1+a_2^m(x,\beta^*)}{2}\cdot\frac{x}{2} \\ +\frac{1-a_2^m(x,\beta^*)}{N_m(\beta^*)}\frac{a_2^m(x,\beta^*)}{2}\cdot\frac{x}{2}\end{array}\right)dx \\
&+\frac{1}{\tau}\int_{x_0}^{1}\left(\begin{array}{c} \frac{1-a_1^m(x,\beta^*)}{N_m(\beta^*)}\left[-\frac{a_1^m(x,\beta^*)}{1-a_1^m(x,\beta^*)}+\frac{1-x^{2m}(\beta^*)-N_m(\beta^*)}{N_m(\beta^*)}\right]\frac{1+a_1^m(x,\beta^*)}{2}.x \\ +\frac{1-a_1^m(x,\beta^*)}{N_m(\beta^*)}\frac{a_1^m(x,\beta^*)}{2}.x\end{array}\right)dx \\
&= \frac{1}{2\tau N_m(\beta^*)}\int_{x^{2m}(\beta^*)}^{x_0}\left[\begin{array}{c} -a_2^m(x,\beta^*)\left(1+a_2^m(x,\beta^*)\right) \\ +\frac{1-x^{2m}(\beta^*)-N_m(\beta^*)}{N_m(\beta^*)}\left(1-\left(a_2^m(x,\beta^*)\right)^2\right) \\ +\left(1-a_2^m(x,\beta^*)\right)a_2^m(x,\beta^*)\end{array}\right]\frac{x}{2}dx \\
&+\frac{1}{2\tau N_m(\beta^*)}\int_{x_0}^{1}\left[\begin{array}{c} -a_1^m(x,\beta^*)\left(1+a_1^m(x,\beta^*)\right) \\ +\frac{1-x^{2m}(\beta^*)-N_m(\beta^*)}{N_m(\beta^*)}\left(1-\left(a_1^m(x,\beta^*)\right)^2\right) \\ +\left(1-a_1^m(x,\beta^*)\right)a_1^m(x,\beta^*)\end{array}\right]xdx \\
&= \frac{1}{2\tau N_m(\beta^*)}\int_{x^{2m}(\beta^*)}^{x_0}\left[\begin{array}{c} \frac{1-x^{2m}(\beta^*)-N_m(\beta^*)}{N_m(\beta^*)} \\ -\frac{1-x^{2m}(\beta^*)+N_m(\beta^*)}{N_m(\beta^*)}\left(a_2^m(x,\beta^*)\right)^2\end{array}\right]\frac{x}{2}dx \\
&+\frac{1}{2\tau N_m(\beta^*)}\int_{x^{2m}(\beta^*)}^{x_0}\left[\begin{array}{c} \frac{1-x^{2m}(\beta^*)-N_m(\beta^*)}{N_m(\beta^*)} \\ -\frac{1-x^{2m}(\beta^*)+N_m(\beta^*)}{N_m(\beta^*)}\left(a_1^m(x,\beta^*)\right)^2\end{array}\right]xdx
\end{aligned}$$

Hence, $\frac{\delta \bar{k}_p^m}{\delta \tau} > 0$ if and only if

$$\begin{aligned}
& \left(1-x^{2m}(\beta^*)-N_m(\beta^*)\right)\left[\int_{x^{2m}(\beta^*)}^{x_0}\frac{x}{2}dx+\int_{x^{2m}(\beta^*)}^{x_0}xdx\right] \\
> & \left(1-x^{2m}(\beta^*)+N_m(\beta^*)\right)\left[\int_{x^{2m}(\beta^*)}^{x_0}\left(a_2^m(x,\beta^*)\right)^2\frac{x}{2}dx+\int_{x^{2m}(\beta^*)}^{x_0}\left(a_1^m(x,\beta^*)\right)^2xdx\right]
\end{aligned}$$

Notice that

$$\int\limits_{x^{2m}(\beta^*)}^{x_0}\frac{x}{2}dx+\int\limits_{x^{2m}(\beta^*)}^{x_0}xdx=\frac{1}{2}-\frac{x_0^2}{4}-\frac{\left(x^{2m}(\beta^*)\right)^2}{4}$$

while

$$\begin{aligned}
N_m(\beta^*) &= 1-x^{2m}(\beta^*)-\left[\int_{x^{2m}(\beta^*)}^{x_0}a_2^m(x,\beta^*)\,dx+\int_{x_0}^{1}a_1^m(x,\beta^*)\,dx\right] \\
&= 1-x^{2m}(\beta^*)-\Gamma
\end{aligned}$$

where

$$\begin{aligned}\Gamma &= \int_{x^{2m}(\beta^*)}^{x_0} a_2^m (x, \beta^*)\, dx + \int_{x_0}^{1} a_1^m(x, \beta^*)\, dx \\ &= \frac{2\tau}{\beta} \log \left(\frac{\beta^* \frac{x_0}{2} - \bar{q}^g}{\beta^* \frac{x^{2m}}{2} - \bar{q}^g} \right) + \frac{\tau}{\beta} \log \left(\frac{\beta^* - \bar{q}^g}{\beta^* x_0 - \bar{q}^g} \right)\end{aligned}$$

The last expression is obtained by using $a_2^m (x, \beta^*)$ and $a_1^m (x, \beta^*)$ from Eqs. (16) and (14) respectively and integrating. Notice that $1 - x^{2m} > \Gamma > 0$, since $x^{2m} < x_0 < 1$ and $a_2^m (x, \beta^*) \leq 1$ and $a_1^m (x, \beta^*) < 1$. Thus,

$$1 - x^{2m} (\beta^*) - N_m (\beta^*) = \Gamma$$

and

$$1 - x^{2m} (\beta^*) + N_m (\beta^*) = 2 \left(1 - x^{2m} (\beta^*)\right) - \Gamma$$

Now integrating

$$\int_{x^{2m}(\beta^*)}^{x_0} (a_2^m (x, \beta^*))^2 \frac{x}{2} dx + \int_{x_0}^{1} (a_1^m (x, \beta^*))^2 x dx$$

and after manipulating some expressions we get

$$\begin{aligned}&\int_{x^{2m}(\beta^*)}^{x_0} (a_2^m (x, \beta^*))^2 \frac{x}{2} dx + \int_{x_0}^{1} (a_1^m (x, \beta^*))^2 x dx \\ = \;& \frac{\tau}{\beta} . \left[\frac{2\tau}{\beta} \log \left(\frac{\beta^* \frac{x_0}{2} - \bar{q}^g}{\beta^* \frac{x^{2m}}{2} - \bar{q}^g} \right) + \frac{\tau}{\beta} \log \left(\frac{\beta^* - \bar{q}^g}{\beta^* x_0 - \bar{q}^g} \right) \right] \\ & + \frac{2\tau^2}{\beta^2} . \bar{q}^g \left[\frac{1}{\beta^* \frac{x^{2m}}{2} - \bar{q}^g} - \frac{1}{\beta^* \frac{x_0}{2} - \bar{q}^g} \right] + \frac{\tau^2}{\beta^2} . \bar{q}^g \left[\frac{1}{\beta^* x_0 - \bar{q}^g} - \frac{1}{\beta^* - \bar{q}^g} \right] \\ = \;& \frac{\tau}{\beta} \Gamma + \frac{\tau}{\beta} \Delta\end{aligned}$$

where

$$\Delta = \frac{2\tau}{\beta} . \bar{q}^g \left[\frac{1}{\beta^* \frac{x^{2m}}{2} - \bar{q}^g} - \frac{1}{\beta^* \frac{x_0}{2} - \bar{q}^g} \right] + \frac{\tau}{\beta} . \bar{q}^g \left[\frac{1}{\beta^* x_0 - \bar{q}^g} - \frac{1}{\beta^* - \bar{q}^g} \right]$$

Hence, using the expressions we derived

$$\frac{\delta \bar{k}_p^m}{\delta \tau} > 0$$

if and only if

$$\frac{\Gamma}{2\left(1-x^{2m}\left(\beta^{*}\right)\right)-\Gamma}\left(\frac{1}{2}-\frac{x_0^2}{4}-\frac{\left(x^{2m}\left(\beta^{*}\right)\right)^2}{4}\right)>\frac{\tau}{\beta}\Gamma+\frac{\tau}{\beta}\Delta$$

Since $x^{2m}\left(\beta^{*}\right)=\frac{\tau+\bar{q}^g}{\frac{\beta}{2}}$, $\frac{\tau}{\beta}=\frac{x^{2m}(\beta^{*})}{2}-\frac{\bar{q}^g}{\beta}$. Therefore, we can rewrite the above inequality as

$$\Gamma\left(\frac{\frac{1}{2}-\frac{x_0^2}{4}-\frac{\left(x^{2m}(\beta^{*})\right)^2}{4}}{2\left(1-x^{2m}\left(\beta^{*}\right)\right)-\Gamma}-\frac{x^{2m}\left(\beta^{*}\right)}{2}\right)>\frac{\tau}{\beta}\Delta-\frac{\bar{q}^g}{\beta}\Gamma \tag{31}$$

Since $\Gamma>0$, the LHS of above is positive if and only if

$$\frac{\frac{1}{2}-\frac{x_0^2}{4}-\frac{\left(x^{2m}(\beta^{*})\right)^2}{4}}{2\left(1-x^{2m}\left(\beta^{*}\right)\right)-\Gamma}>\frac{x^{2m}\left(\beta^{*}\right)}{2}$$

$$\Leftrightarrow 1-\frac{x_0^2}{2}-\frac{\left(x^{2m}\left(\beta^{*}\right)\right)^2}{2}-2\left(1-x^{2m}\left(\beta^{*}\right)\right)x^{2m}\left(\beta^{*}\right)+\Gamma x^{2m}\left(\beta^{*}\right)>0$$

$$\Leftrightarrow \left(1-x^{2m}\left(\beta^{*}\right)\right)^2+\frac{\left(x^{2m}\left(\beta^{*}\right)\right)^2}{2}+\Gamma x^{2m}\left(\beta^{*}\right)-\frac{x_0^2}{2}>0$$

We write the LHS of the last inequality as $L\left(x^{2m}\right)$. Notice that $\lim_{x^{2m}\to x_0} L\left(x^{2m}\right)>0$. Also notice that

$$\begin{aligned}
L'\left(x^{2m}\right) &= -2\left(1-x^{2m}\right)+x^{2m}+x^{2m}\frac{\delta\Gamma}{\delta x^{2m}}+\Gamma \\
&= -2\left(1-x^{2m}\right)+x^{2m}-x^{2m}+\Gamma \\
&= -2\left(1-x^{2m}\right)+\Gamma \\
&< 0
\end{aligned}$$

since $\frac{\delta\Gamma}{\delta x^{2m}}=-1$ and $\Gamma<\left(1-x^{2m}\right)$. Hence, $L\left(x^{2m}\right)>0$ for all $x^{2m}\leq x_0$.

The RHS of Eq. (31) can be written as

$$
\begin{aligned}
& \frac{\tau}{\beta}\Delta-\frac{\bar{q}^g}{\beta}\Gamma \\
= \;& \frac{\tau}{\beta}\left(\frac{2\tau}{\beta}.\bar{q}^g\left[\frac{1}{\beta^*\frac{x^{2m}}{2}-\bar{q}^g}-\frac{1}{\beta^*\frac{x_0}{2}-\bar{q}^g}\right]+\frac{\tau}{\beta}.\bar{q}^g\left[\frac{1}{\beta^* x_0-\bar{q}^g}-\frac{1}{\beta^*-\bar{q}^g}\right]\right) \\
& -\frac{\bar{q}^g}{\beta}\left(\frac{2\tau}{\beta}\log\left(\frac{\beta^*\frac{x_0}{2}-\bar{q}^g}{\beta^*\frac{x^{2m}}{2}-\bar{q}^g}\right)+\frac{\tau}{\beta}\log\left(\frac{\beta^*-\bar{q}^g}{\beta^* x_0-\bar{q}^g}\right)\right) \\
= \;& \frac{\tau\bar{q}^g}{\beta^2}.2\left[\left(\frac{\tau}{\beta^*\frac{x^{2m}}{2}-\bar{q}^g}+\log\frac{\beta^*\frac{x^{2m}}{2}-\bar{q}^g}{\tau}\right)-\left(\frac{\tau}{\beta^*\frac{x_0}{2}-\bar{q}^g}+\log\frac{\beta^*\frac{x_0}{2}-\bar{q}^g}{\tau}\right)\right] \\
& +\frac{\tau\bar{q}^g}{\beta^2}.\left[\left(\frac{\tau}{\beta^* x_0-\bar{q}^g}+\log\frac{\beta^* x_0-\bar{q}^g}{\tau}\right)-\left(\frac{\tau}{\beta^*-\bar{q}^g}+\log\frac{\beta^*-\bar{q}^g}{\tau}\right)\right] \\
= \;& \frac{\tau\bar{q}^g}{\beta^2}.2\left[\left(a_2^m\left(x^{2m},\beta^*\right)+\log\frac{1}{a_2^m\left(x^{2m},\beta^*\right)}\right)-\left(a_2^m\left(x_0,\beta^*\right)+\log\frac{1}{a_2^m\left(x_0,\beta^*\right)}\right)\right] \\
& +\frac{\tau\bar{q}^g}{\beta^2}.\left[\left(a_1^m\left(x_0,\beta^*\right)+\log\frac{1}{a_1^m\left(x_0,\beta^*\right)}\right)-\left(a_1^m\left(1,\beta^*\right)+\log\frac{1}{a_1^m\left(1,\beta^*\right)}\right)\right]
\end{aligned}
$$

Since the function $\frac{1}{y}+\log y$ is rising in y for $y>1$, $\frac{x^{2m}}{2}<\frac{x_0}{2}<x_0<1$ and both $a_2^m\left(x,\beta^*\right)$ and $a_1^m\left(x,\beta^*\right)$ are falling in x, both bracketed terms in the last line of the above are negative. Thus,

$$\frac{\tau}{\beta}\Delta-\frac{\bar{q}^g}{\beta}\Gamma<0$$

This shows that

$$\frac{\delta\bar{k}_p^m}{\delta\tau}>0$$

Since girls face a higher τ than boys,

$$\bar{k}_p^f>\bar{k}_p^m$$

This completes the proof.

4.3 Proof of Proposition 10

The extent of loss for a boy of ability a from being matched with a male teacher instead of a female teacher is $k_m^F(a)-k_m^M(a)$ and for a girl of same ability is $k_f^F(a)-k_f^M(a)$. We show that

$$k_m^F(a)-k_m^M(a)<k_f^F(a)-k_f^M(a)$$

for $a \geq a_2^f(x_0, \beta^*)$, i.e when both boys and girls are taught by teachers of both genders.

Notice that

$$\begin{aligned} k_m^F(a) - k_m^M(a) &= \frac{a}{2}\left[\frac{1 - \frac{3x_0^2}{4} - \frac{(x_m(a))^2}{4}}{1 - \frac{x_0}{2} - \frac{x_m(a)}{2}} - \frac{x_m(a) + x_0}{2}\right] \\ &= \frac{a}{2}.\frac{2 - x_0^2 - x_0 - x_m(a)(1 - x_0)}{1 - \frac{x_0}{2} - \frac{x_m(a)}{2}} \\ &= \frac{a}{2}.\frac{[2 + x_0 - x_m(a)](1 - x_0)}{1 - \frac{x_0}{2} - \frac{x_m(a)}{2}} \end{aligned}$$

and similarly

$$k_f^F(a) - k_f^M(a) = \frac{a}{2}.\frac{[2 + x_0 - x_f(a)](1 - x_0)}{1 - \frac{x_0}{2} - \frac{x_f(a)}{2}}$$

Now,

$$k_m^F(a) - k_m^M(a) < k_f^F(a) - k_f^M(a)$$

if and only if

$$\frac{2 + x_0 - x_m(a)}{1 - \frac{x_0}{2} - \frac{x_m(a)}{2}} < \frac{2 + x_0 - x_f(a)}{1 - \frac{x_0}{2} - \frac{x_f(a)}{2}}$$

Cross-multiplication and canceling terms from both sides will reduce the inequality to

$$x_m(a) < x_f(a)$$

which holds for the range of a we consider here.

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